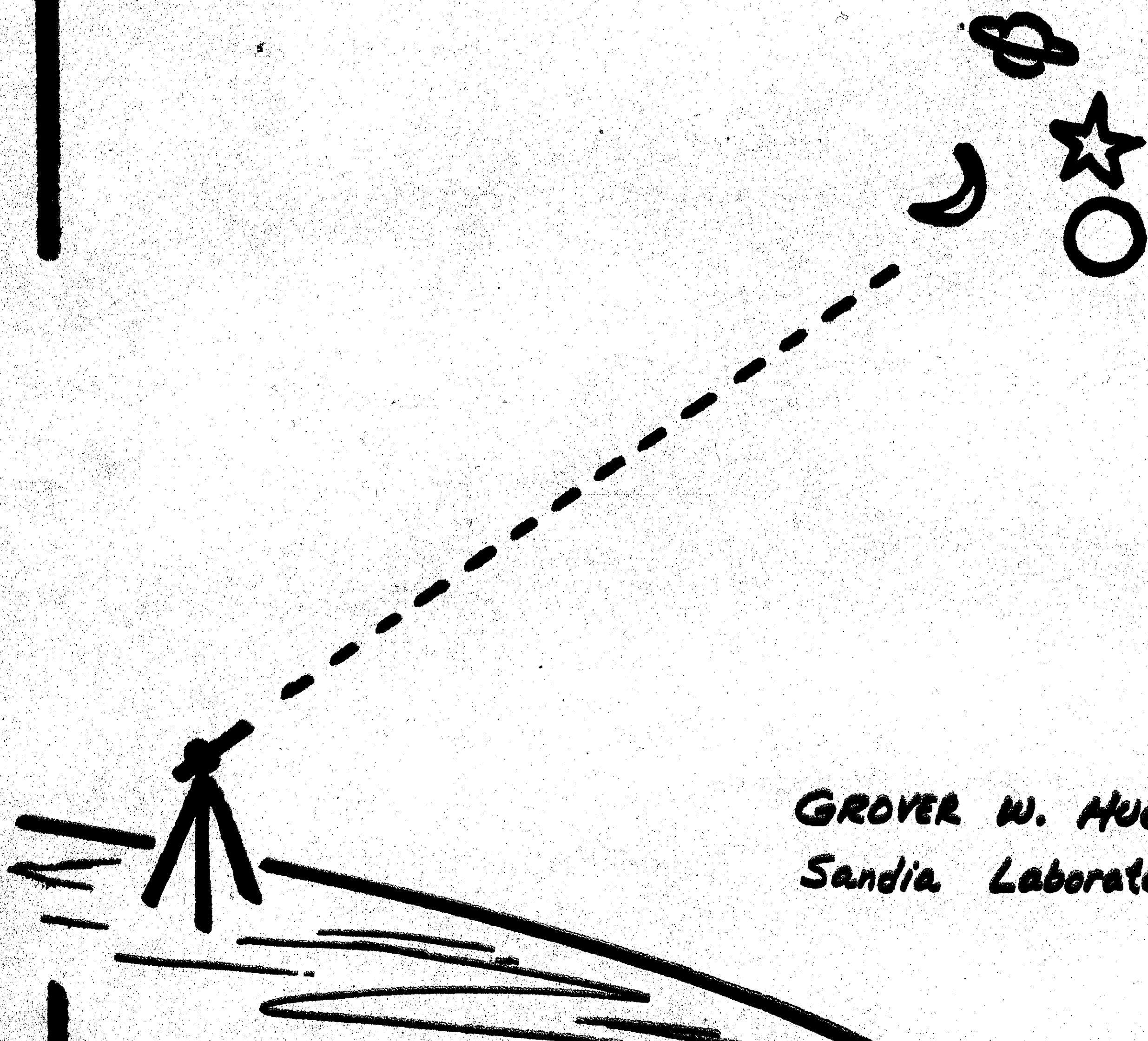
BOYSON

ENGINEERING ASTRONOMY



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ENGINEERING ASTRONOMY

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6-1728

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Albuquerque, N.M.

1985 February 1

FOREWORD

The name "Engineering Astronomy" refers to that blend of selected topics from the closely-related disciplines of positional astronomy, astrometry, and celestial mechanics which I have found necessary for the solution of certain problems with which I have been confronted over the past quarter-century or so as a practicing mechanical design engineer. The material contained in this text is intended to introduce the student to the fundamental nomenclature and methods of these disciplines, as applied to engineering problems at Sandia National Laboratories. Since we at Sandia are not primarily, if indeed at all, interested in the navigational aspects of astronomy, references to sextants, dip of the horizon, and like matters are omitted; the discussion of observing instruments is limited to transits, theodolites, and tracking telescopes which we would expect to use in the field.

In addition to the text, there are several other publications which we will find necessary to consult for data required in the solution of various problems. These include:

(a) "The Astronomical Almanac," an annual publication of the Nautical Almanac Office, United States Naval Observatory, Washington, D.C. This volume, commonly referred to as simply "the Almanac," or abbreviated as "A.A.," is the basic reference source for fundamental constants, data, and methods of computation for our class. Until 1981, this publication was entitled "The American Ephemeris and Nautical Almanac," often called "the Ephemeris" for short, and abbreviated as "A.E." The student should bear this relatively recent name change in mind, as references to the older title appear in many related publications, including the present text.

(b) "The Nautical Almanac," another and quite different publication issued by the same office as above. This volume, which is often abbreviated as "N.A.," contains material drawn from the Astronomical Almanac but presented in a form to render it more readily usable by navigators of ships at sea. We shall use portions of it, but we shall depend upon the Astronomical Almanac to meet the most of our needs.

(c) "Explanatory Supplement to the Ephemeris," copyright 1961, which gives detailed explanations of the tables in the Astronomical Almanac, sample calculations to illustrate their use, and a considerable amount of information on related topics such as the calendar.

(d) "Apparent Places of Fundamental Stars," issued on an annual basis by the Astronomisches Rechen-Institut at Heidelberg, containing the positions of over 1,500 stars given at intervals of ten days or less during the current year.

(e) various star charts, atlases, and catalogs.

It would certainly be best to provide each student with his own copy of each of the above publications, but high cost and long delivery times preclude this approach. Instead, necessary portions of the above materials will be reproduced and handed out in class from time to time so that each of you will gradually accumulate a number of pages, about 150 to 200 in all, which should be filed in a manner suitable for rapid information retrieval as well as for ease of carrying. This material should be brought to each class meeting for possible reference use.

The data tabulated in the Almanac and in "Apparent Places" are given to a precision commensurate with that required for first-order survey work or for observatory use; the data in the Nautical Almanac are given to a much lower precision, typically to the nearest tenth of a minute of arc, which is usually good enough for the navigator and in many cases for the land surveyor. For class problems, a general rule is to carry out all calculations to the same degree of precision as that of the tabular data, so as to make it easy to check your answer and method.

While it is my hope that the knowledge you will gain in this course will enable you to attack and solve such problems in astronomy as may come to be part of your work at the Laboratories, I will be happy to consult with you on any such problem either during this "semester" or thereafter.

Grover W. Hughes
1985 February 1

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PART I

FUNDAMENTAL PRINCIPLES

CHAPTER 1

INTRODUCTION

1.1 Practical Astronomy. "Practical Astronomy" is traditionally defined as that branch of astronomy which deals with the theory and use of astronomical instruments, methods of observing, the reduction of observations, and the prediction of future positions of the various heavenly bodies. The term "Engineering Astronomy" will be taken to mean that part of practical astronomy in which we here at Sandia Laboratory are interested, concerning the determining of time, latitude, longitude, and azimuth, together with the circumstances of solar eclipses, the apparent track of missiles and artificial satellites, and the positions of the heavenly bodies at any time.

1.2 Heavenly Bodies. Astronomy in general includes the study and description of different bodies such as the sun and the moon that are commonly known as heavenly bodies. These are:

The stars, which are extremely remote, but immense, bodies giving off light and heat by means of their internal thermonuclear processes, and which appear to shine in the sky as points of light of various brightnesses. To the unaided eye not more than about 2,500 are visible at one time, but it has been estimated that the 5 m (200 inch) telescope on Mt. Palomar could reveal over 1 billion. The stars (with the exception of our own sun) are so far away that we are hardly able to detect their relative motions, and for this reason they are often called fixed stars.

The sun, which is a star without which life on the earth, at least as we know it, would be impossible. It is an average-size star about 1,392,000 kilometers (865,000 miles) in diameter with a mass 333,000 times that of the earth.

The planets, nine in number, which are opaque spheroidal bodies revolving about the sun in orbits which are ellipses with the sun at one of the foci. The orbits of the planets are nearly in the same plane. As viewed from a far-off northern point in space, the planets move about the sun in a counter-clockwise direction. They shine by reflected light from the sun and to the unaided eye look much like stars.

Natural satellites, which in physical constitution resemble the planets and revolve about them in elliptic orbits; the moon is the natural satellite of the earth.

Comets, which are bodies of small mass and very low density compared with the planets, revolving about the sun in elliptic or parabolic orbits. The radiation from the sun makes them shine. Bright comets appear in the sky as hazy spots with tails of pale light streaming from them.

Asteroids, also known as planetoids, which are small and usually irregularly shaped solid bodies revolving about the sun in independent orbits lying mainly between those of Mars and Jupiter. About 50,000 of these objects have been discovered; orbits of about 1,800 of them have been determined. The asteroids vary in size from about 1 kilometer to about 760 kilometers in diameter.

Table 1.1 Orbital Data of the Planets
(A.A. 1985, p. E3)

Planet	Symbol	Mean distance from sun			Equatorial Diameter		Time for 1 revolution around sun	Orbital eccentricity	Orbital inclination	Number of satellites
		A.U.	10^6 km	10^6 mi.	km	miles				
Mercury	☿	0.39	58	36	4,878	3,131	88 days	0.206	7°.00	0
Venus	♀	.72	108	67	12,104	7,521	225 "	.007	3.39	0
Earth	⊕	1.00	150	93	12,756	7,926	365 "	.017	0	1
Mars	♂	1.52	228	142	6,794	4,222	687 "	.093	1.85	2
Jupiter	♃	5.20	778	484	142,796	88,730	11.9 yrs	.048	1.30	16
Saturn	♄	9.56	1,426	887	120,000	74,565	29.5 "	.057	2.48	15
Uranus	⛢	19.3	2,870	1,786	50,800	31,566	84.0 "	.047	0.77	5
Neptune	♆	30.3	4,496	2,797	48,600	30,200	164.8 "	.007	1.79	2
Pluto	♇	39.6	5,900	3,675	5,000	3,100	248.4 "	.252	17.13	1

The sun (astronomical symbol ⊙), the planets with their satellites, the comets, and the asteroids form the solar system. Some characteristics of the planets are given in Table 1.1.

In addition to the natural bodies described above, we must also consider man-made objects which, although very much smaller and less massive than most of the other natural bodies, may appear as star-like or meteor-like. These include:

Artificial satellites of the earth, around which they revolve in elliptic orbits of various sizes and with the earth at one of the foci. About 5,000 such objects were in orbit as of 1982 January 1.

Missiles and rockets whose lifetimes are relatively short, being launched from the surface of the earth, possibly leaving the atmosphere and reentering, with ground impact occurring less than one earth circumference away.

1.3 The earth is the third planet from the sun and the fifth largest of the nine planets. Its shape is nearly that of an oblate spheroid, rotating on its shorter (polar) axis in a counterclockwise direction as seen from a far-off northern point in space. The Astronomical Almanac contains, in Section K, a number of fundamental constants pertaining to the earth and its figure (shape). A few of these constants are:

(a) the equatorial radius is 6,378.140 km
(b) the flattening reciprocal is 298.257
(c) mean distance from the sun is 149,597,870 km

In addition, the earth revolves about the sun in an orbit which is an ellipse having the sun at one of the foci; the direction of this revolution is, like its rotation, counterclockwise as viewed from a remote northern point in space. The average orbital speed is about 30 km/sec. About January 2 of each year the earth is at perihelion (the point in its orbit which is nearest the sun) and its orbital speed is a maximum; about July 2 it is at aphelion (farthest from the sun) and its speed is a minimum (see Fig. 1.1).

The orbital plane of the earth is called the plane of the ecliptic, taking its name from the fact that eclipses can occur only when the moon is on or very near this plane. The equatorial plane of the earth is tipped at an angle of about $23\frac{1}{2}°$ to the ecliptic plane; this angle is formally known as the obliquity of the ecliptic. Daily values of this angle are given in the "Nutation, Obliquity, Day Numbers" table in Section B of the Astronomical Almanac.

The earth's rotational (polar) axis is tipped from the perpendicular to the ecliptic plane by the same angle. For the present discussion, the earth's axis will be considered to point always in the same direction in space, which is very nearly true, since the real motions of the axis are very slow, with the effects of such motions not being noticed by the casual observer for many decades, if at all. Referring to Fig. 1.1 with the above assumption in mind, it will be seen that when the earth is at A, the sun shines vertically downward on points $23\frac{1}{2}°$ north of the earth's equator. This occurs about June 21 of each year and marks the beginning of summer for the Northern Hemisphere. About

September 21, the earth is at B, the sun shines vertically downward on points on the equator, and oblique rays just reach the north and south poles. This position marks the beginning of autumn. Three months later, the earth is at C, where conditions are opposite to those at A, that is, the sun shines vertically downward on points about 23½° south of the equator. This occurs about December 21 and marks the beginning of winter for the Northern Hemisphere. On March 21, the beginning of spring, the north and south poles again just receive light as they did at B.

The exact instants of time, or epochs, corresponding to the points just discussed are called the summer solstice, the autumnal (fall) equinox, the winter solstice, and the vernal (spring) equinox, respectively. The average dates are as given above; the exact values for the current year are given in Section A of the Astronomical Almanac.

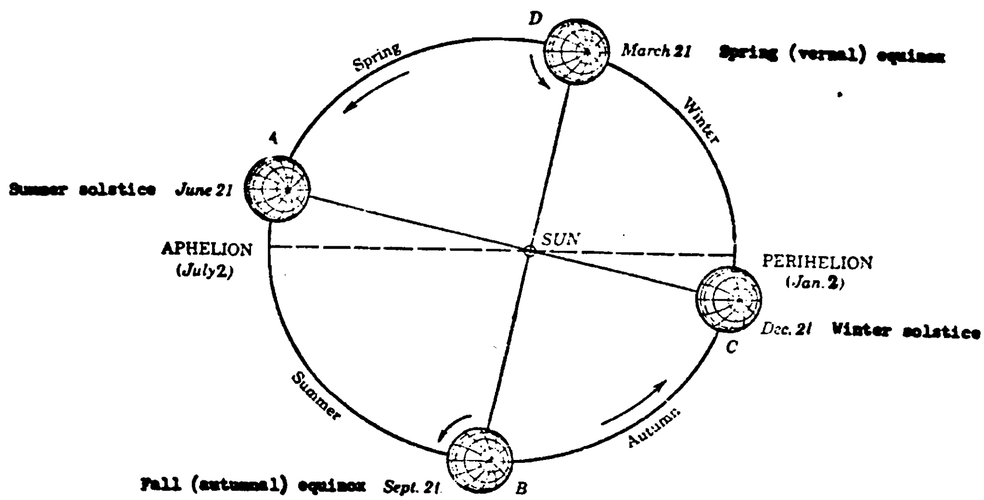


Fig. 1.1 The seasons.

1.4 The Moon. The diameter of the moon is about 3,480 km (2,160 miles), somewhat greater than one-fourth of the diameter of the earth about which it revolves in an elliptical path in 27 1/3 days sidereal period (referred to the star field) or 29½ days synodic period (referred to the earth-sun line). The plane of the orbit of the moon about the earth is inclined to the plane of the earth's orbit about the sun (the ecliptic) at an angle of about 5°. While the earth is revolving about the sun, the moon is revolving about the earth in the same direction at an average distance of about 384,000 km (239,000 miles).

Like the planets, the moon shines by reflected light of the sun. When the moon, during its revolution about the earth, is between us and the sun, or nearly so, its dark hemisphere is toward us and the moon is invisible. This phase is known as the new moon. About one week later, at first quarter, the moon has moved ¼ of the way around the earth, so that half of the sunlit hemisphere is visible from the earth; and two weeks later, when the earth is between the sun and the moon, the entire sunlit hemisphere of the moon is facing the earth and we have full moon. Third quarter occurs about a week later than this, when again only half the sunlit hemisphere is visible.

The times corresponding to the various phases of the moon are given in the Almanac, in Section A, "Phenomena".

1.5 The Celestial Sphere. As we look at the heavens on a clear night, the stars appear to be fixed on the inner surface of a vast sphere known as the celestial sphere and we appear to be at the center of this sphere. In reality, the stars are scattered in space, and we, in their midst, project their images on this imaginary sphere. The stars are so remote from the observer that the celestial sphere is assumed infinite in radius, with its center at the observer on the surface of the earth, or at the center of the earth, or at the center of the sun.

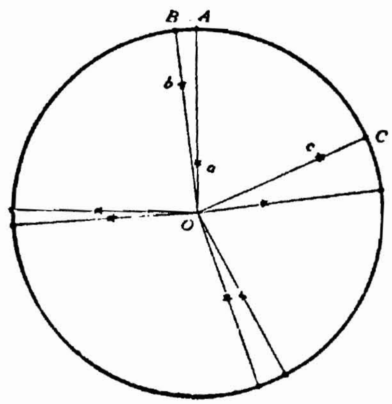
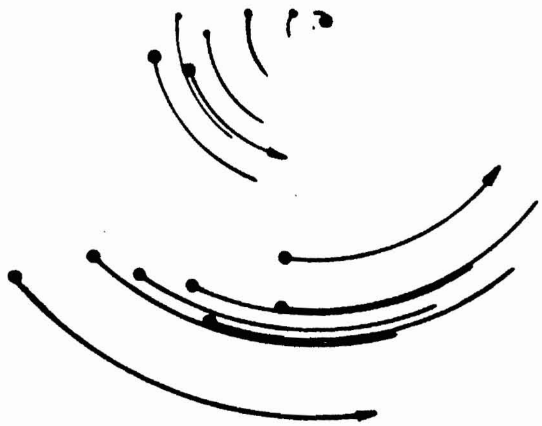


Fig. 1.2 Apparent positions of the heavenly bodies. To the observer at O, the heavenly bodies a, b, and c appear on the celestial sphere at A, B, and C, respectively; a and b appear very close to each other, though in reality they are separated by a vast distance.

After watching the sky for some time, we see that some stars have disappeared below the western horizon and others have appeared above the eastern horizon, but the relative positions of the stars visible remain the same. Hence we conclude that the celestial sphere apparently rotates on an axis. This apparent rotation of the celestial sphere, making stars rise in the east and set in the west, is due to the actual rotation from west to east of the earth on its axis. The celestial poles are the two points where the axis of rotation of the earth, extended, pierces the celestial sphere. Each star appears to describe a circle having its center on the line joining the celestial poles; these circles are known as diurnal circles. Diurnal circles are illustrated in Fig. 1.3. Work in practical astronomy is immensely simplified by making use of the apparent rotation of the celestial sphere in preference to the actual rotation of the earth.

Fig. 1.3 Diurnal circles traced by the Big Dipper and the Little Dipper. The initial positions of the stars are indicated by the dots; arrows indicate the direction of apparent motion.



1.6 Apparent Path of the Sun among the Stars. The circle KLM in Fig.1.4 represents the intersection of the celestial sphere with the plane of the earth's orbit ABC. Let K, L, and M be the projections of stars on the celestial sphere. S' is the projection of the sun S on the celestial sphere when the earth is at A. Twenty-four sidereal hours later, the earth will be at a position such as B, the stars will appear in the same position as before, but the projection of the sun will be S". Hence, because of the motion of the earth in its orbit about the sun, the sun appears to move among the stars from west to east. The ecliptic is the intersection of the plane of the earth's orbit with the celestial sphere, or the great circle described by the sun in its apparent motion during the year.

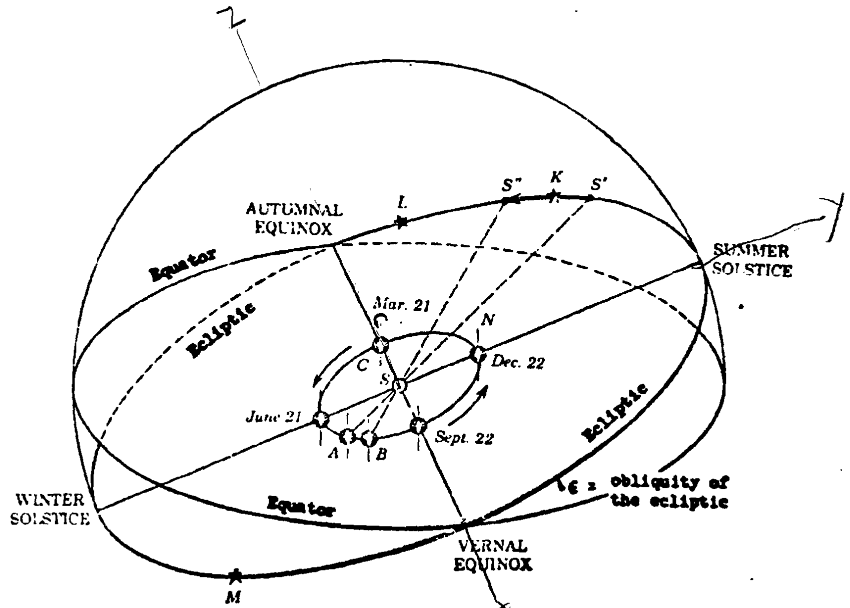


Fig. 1.4 The sun projected on the celestial sphere. As the earth moves from *A* to *B*, the sun appears to move from *S'* to *S''*. This motion is toward the *east* and about 1° *per day*.

Since the earth completes one revolution in about $365\frac{1}{4}$ days, the apparent motion of the sun among the stars is about 1° per day. This motion is illustrated in Fig. 1.5, which is a portion of a star chart with the sun's position shown for three different days.

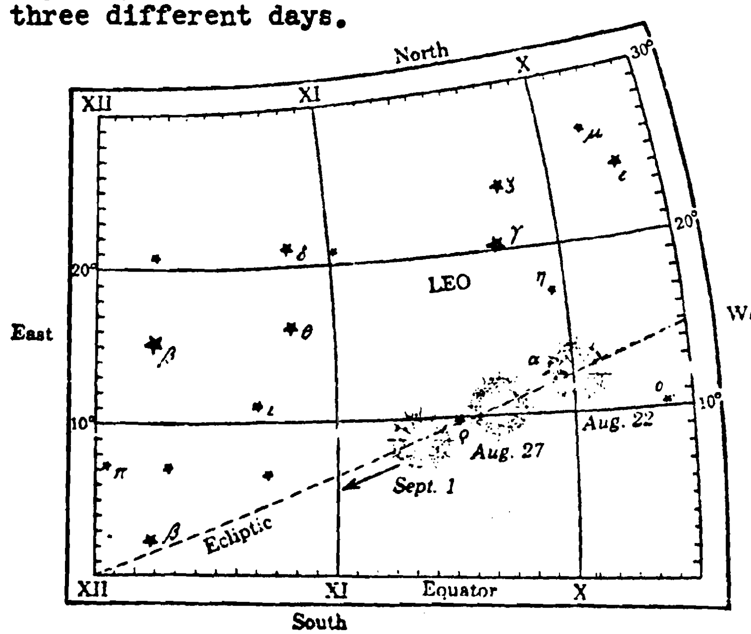


Fig. 1.5 Apparent motion of the sun among the stars. The sun has moved in the interval of 10 days about 10° toward the *east*

1.7 Apparent Path of the Moon and Planets among the Stars. Owing to the rotation of the earth on its axis the moon rises and sets like the rest of the heavenly bodies. But the moon revolves about the earth and hence moves in an easterly direction at about 13° per day (360 ÷ 27.3). Thus it rises later each day. Just as the sun appears to move among the stars about 1° per day, the moon appears to move about 13° in the same direction. This motion relative to the stars is shown in Figs. 1.6a and 1.6b.

Fig. 1.6a Appearance of the western sky at sunset, on a particular day, in lat. 40°N. The moon is 2 days old and is about 26° from the sun. The Pleiades are shown above the sun; Aldebaran is just below the moon; the constellation Orion is to the left.

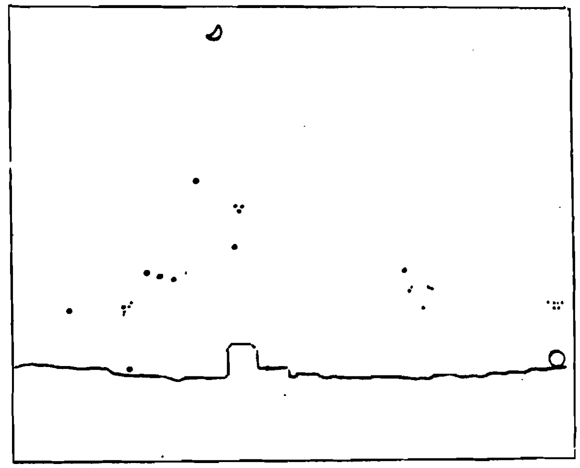


Fig. 1.6b Appearance of the western sky at sunset, 2 days later. The star configurations remain the same in the two figures, but the stars are nearer the western horizon, which shows the apparent easterly motion of the sun with respect to the stars. This is more apparent in the case of the moon.

The apparent motion of the planets on the celestial sphere is somewhat analogous to the motion of the sun and the moon but is more irregular. The movement of the earth around the sun is combined with that of the planet in such a way that sometimes the planet appears to move westward among the stars.

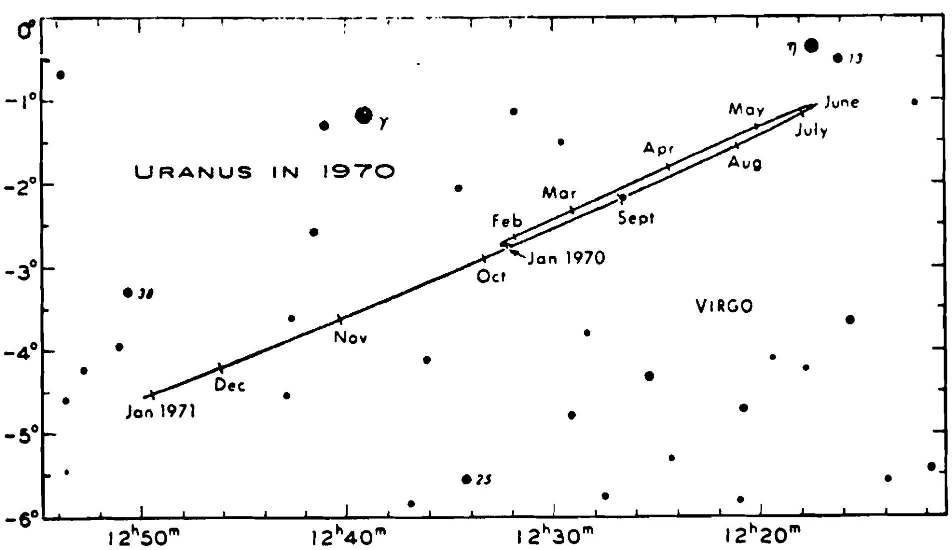


Fig. 1.7 The chart above shows the path of Uranus in Virgo, with stars plotted to about magnitude $7\frac{1}{2}$; north is at the top, to agree with the view in binoculars.

1.8 Constellations. The stars as they appear on the celestial sphere have been divided into groups that are known as constellations. At present, the entire surface of the celestial sphere is divided into 88 areas, or constellations.

Most of the names of the constellations come to us from the ancients. Many names are those of animals; others represent characters in Greek mythology. The stars in a constellation are designated by letters of the Greek alphabet. Usually the brightest star in the constellation receives the letter α, the second brightest β, and so on. For example, in the constellation Ursa Minor (the Little Bear), the North Star is the brightest and is named α Ursae Minoris (the genitive case being used); the second brightest is β Ursæ Minoris. A few prominent stars have individual names; for example, α Lyræ is known as Vega, and β Orionis as Rigel. When the naked-eye stars of a constellation are so numerous as to exhaust the letters of the Greek alphabet, the Roman letters are used. It is apparent that this method will fail in the case of telescopic stars. In this case, a star is referred to by its number in some catalogue. For example, GC1353 means the star so numbered in Boss's "General Catalogue".

Fig. 1.8 shows a familiar grouping of stars commonly called the "Big Dipper." which is actually just a portion of the constellation Ursa Major, the Great Bear; the complete constellation includes even more stars than are shown here.

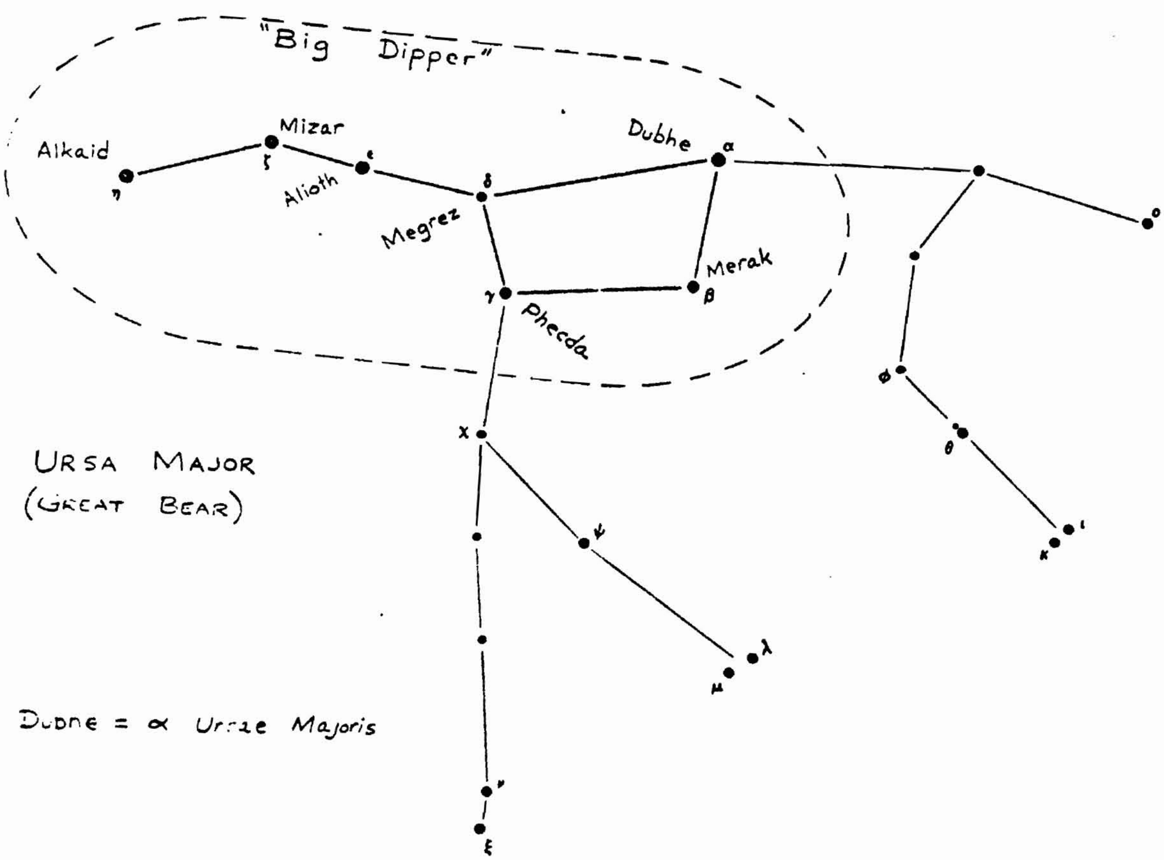


Fig. 1.8

1.9 Magnitude of Stars. Inasmuch as distances of the stars from the observer are different and their intrinsic brightness is different, their apparent brightness is different. To classify the stars according to their brightness, the ancient astronomers adopted an arbitrary scale known as "magnitude" of stars. A modified form of this scale used at present may be explained as follows:

A star of fifth magnitude is $\sqrt[5]{100}$, or 2.512, times as bright as a star of sixth magnitude.

A star of fourth magnitude is $(\sqrt[5]{100})^2$, or 6.31 times as bright as a star of sixth magnitude.

A star of third magnitude is $(\sqrt[5]{100})^3$, or 15.85, times as bright as a star of sixth magnitude.

A star of second magnitude is $(\sqrt[5]{100})^4$, or 39.81, times as bright as a star of sixth magnitude.

A star of first magnitude is 100 times as bright as a star of the sixth magnitude.

The scale may be extended above and below the limits given. That is, a star of zero magnitude is 2.512 times as bright as a star of first magnitude. Vega is about zero magnitude. Fractional and negative magnitudes may likewise be introduced. Table 1.2 gives apparent visual magnitudes of a few celestial objects, for comparison; the "Bright Stars" table in Section H of the Almanac lists 1,475 stars and their visual magnitudes.

Table 1.2 Visual Magnitudes of a Few Heavenly Bodies

Sun	-26.7	Uranus	+7
Moon (full)	-12.7	Neptune	+8
Mercury[1]	-1	Pluto[3]	+15
Venus[2]	-4.5		
Mars	-2	Sirius	-1.46
Jupiter	-2	Vega	+0.03
Saturn	0	Polaris	+2.02

The faintest star which can be seen with 7X50 binoculars is about +9.
The faintest star which can be seen with the 200-inch Hale telescope on Mount Palomar is about +23.

1 the magnitude of a planet varies with its phase and distance; values given are for the brightest apparition.
2 can be seen in daylight with the unaided eye.
3 requires about a 14-inch telescope.

The apparent visual magnitudes of two bodies are related by the expression

$$m_b - m_f = -2.5 \log_{10} \frac{B_b}{B_f} \tag{1-1}$$

where
- m = magnitude
- B = brightness
- b = subscript denoting the brighter body
- f = subscript denoting the fainter body

Example: An artificial earth satellite passes near a star whose visual magnitude is known to be +3.2; the star is estimated to be twice as bright as the satellite. The approximate visual magnitude of the satellite is then calculated as follows:

$$m_b - m_f = -2.5 \log_{10} \frac{B_b}{B_f}$$

$$m_* - m_{sat} = -2.5 \log_{10} \frac{B_*}{B_{sat}}$$

$$+3.2 - m_{sat} = -2.5 \log_{10} 2$$

$$+3.2 - m_{sat} = -2.5(0.30103) = -0.75 \approx -0.8$$

$$m_{sat} = +3.2 + 0.8$$

$$m_{sat} = +4.0$$

1.10 Units of Angular Measurements. The apparent separation of one heavenly body from another is usually measured on the celestial sphere in units of degrees, minutes, and seconds of arc, denoted by the symbols ° ' " respectively. For example, referring to Fig. 1.8 we say that the distance between the "pointer stars" Merak and Dubhe is approximately 5°. (The real distance between them is disregarded; see Fig. 1.2 .)

Another system of angular measurement which is very convenient in astronomy is that of hours, minutes, and seconds of time, denoted by the symbols h m s respectively; in this system, the circle is considered to be divided into 24 units called hours.

The relation between the two systems is as follows:

1^h corresponds to 15°
1^m corresponds to 15'
1^s corresponds to 15"

1° corresponds to 4^m
1' corresponds to 4^s
1" corresponds to $0\overset{s}{.}067$

Tables B-1 and B-2 in the Appendix permit rapid conversion from one system to the other.

Example 1: Convert 55° 40' 44".6 to time units.

From Table B-2 in the Appendix , the following correspondence of values is obtained:

55°	is equivalent to	3^h 40^m
40'	is equivalent to	2^m 40^s
44"	is equivalent to	$2\overset{s}{.}933$
".6	is equivalent to	$\overset{s}{.}040$
55° 40' 44".6	is equivalent to	3^h 42^m $42\overset{s}{.}973$

Example 2: Convert 17^h 27^m $43\overset{s}{.}74$ to angular units.

From Table B-1 in the Appendix, the following correspondence of values is obtained:

12^h	is equivalent to	180°
5^h 27^m	is equivalent to	81° 45'
43^s	is equivalent to	10' 45"
$\overset{s}{.}74$	is equivalent to	11".10
17^h 27^m $43\overset{s}{.}74$	is equivalent to	261° 55' 56".10

Exercises

1-1. Calculate the radius of the sun in kilometers, using the semidiameter and true distance (given in Astronomical Units) as tabulated in the Almanac for:

- (a) Feb. 13
- (b) June 10
- (c) Oct. 31

Compare the results with the value given on page 1 of the text. (Note: the value of the A.U. in km is given in the Almanac, on page K6. other necessary data are given in the Almanac tables of the Sun, Section C.

1-2. Referring to Table 1.1 of the text, verify the mean distance of

(a) Mercury	(e) Saturn
(b) Venus	(f) Uranus
(c) Mars	(g) Neptune
(d) Jupiter	(h) Pluto

from the sun (in kilometers).

1-3. Find the mean time required for a radio transmission to travel from the earth to the sun; necessary data are given in the Almanac.

1-4. Using the earth's equatorial radius and flattening factor from page K6 of the Almanac, calculate the polar radius.

1-5. Give the dates in the current year corresponding to:

(a) perihelion	(e) summer solstice
(b) aphelion	(f) winter solstice
(c) vernal equinox	(g) Easter
(d) autumnal equinox	(h) full moon in September

1-6. On what dates in the current year is the sun on:
(a) the ecliptic? (b) the celestial equator?

1-7. On the average, what is the daily difference in the time of moon rise, and is it earlier or later each day?

1-8. The star Vega (α Lyrae) has a measured brightness of 416 units on a certain photometric scale. On the same scale, a nearby star has a brightness of 173 units. Give the visual magnitude of this second star.

1-9. On the same scale as that described in the exercise preceding, what would be the expected brightness of the star ϵ Scuti (3rd entry following Vega)?

1-10. Express each of the following angular measurements in the opposite system of units:

(a) $4^h\ 22^m\ 16^s$ = 65° 34′ 0″	(e) $8^h\ 8^m\ 11^s$ = 122° 2′ 45″
(b) 166° 37′ 14″ = 11h 6m 28.933s	(f) $22^h\ 23^m\ 24^s$ = 335° 51′ 0″
(c) 45° 20′ = 3h 1m 20s	(g) 346° 2′ 50″ = 23h 4m 11.333s
(d) $19^h\ 21^m\ 47^s$ = 290° 26′ 45″	(h) 69° 58′ 46″ = 4h 39m 55.067s

Exercises

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Compare the results with the value given on page 1 of the text. (Note: the value of the A.U. in km is given in the Almanac, on page K6. other necessary data are given in the Almanac tables of the Sun, Section C.

1-2. Referring to Table 1.1 of the text, verify the mean distance of

☿ (a)	Mercury 58 (57.909)	♄ (e)	Saturn 1,426
♀ (b)	Venus 108	♅ (f)	Uranus 2870
♂ (c)	Mars 228	♆ (g)	Neptune 4496
♃ (d)	Jupiter 778	♇ (h)	Pluto 5900

from the sun (in kilometers).

1-3. Find the mean time required for a radio transmission to travel from the earth to the sun; necessary data are given in the Almanac. K6 - c
499 sec = 8 min 19 sec

1-4. Using the earth's equatorial radius and flattening factor from page K6 of the Almanac, ~~verify the value given there for~~ CALCULATE the polar radius.
6356.755 km 21.4 m difference

1-5. Give the dates in the current year corresponding to:

(a)	perihelion 1-3 - 20h	(e)	summer solstice 6-21 10h 44m
(b)	aphelion 7-5 10	(f)	winter solstice 12-21 22 08
(c)	vernal equinox 3-20 16 14m	(g)	Easter 4-7
(d)	autumnal equinox 9-23 2 07	(h)	full moon in September 29-00h 08m

1-6. On what dates in the current year is the sun on:
(a) the ecliptic? always (b) the celestial equator? equinox's

1-7. On the average, what is the daily difference in the time of moon rise, and is it earlier or later each day? 50 min

1-8. The star Vega (α Lyrae) has a measured brightness of 416 units on a certain photometric scale. On the same scale, a nearby star has a brightness of 173 units. Give the visual magnitude of this second star.

1-9. On the same scale as that described in the exercise preceding, what would be the expected brightness of the star ε Scuti (3rd entry following Vega)?

1-10. Express each of the following angular measurements in the opposite system of units:

(a)	$4^h\ 22^m\ 16^s$	(e)	$8^h\ 8^m\ 11^s$
(b)	166° 37' 14"	(f)	$22^h\ 23^m\ 24^s$
(c)	45° 20'	(g)	346° 2' 50"
(d)	$19^h\ 21^m\ 47^s$	(h)	69° 58' 46"

CHAPTER 2

SPHERICAL COORDINATE SYSTEMS

2.1 Terminology and Definitions. A sphere may be defined as a closed surface every point of which is equidistant from a fixed point within called the center. A radius of the sphere is any straight line drawn from the center to the surface. A diameter of the sphere is the line formed by any two radii which are oppositely directed. A circle is the intersection of the spherical surface with any plane which cuts that surface; when the cutting plane passes through the center of the sphere, a great circle is formed; a small circle is formed when the cutting plane does not pass through the center of the sphere. The axis of the circle is the line which is perpendicular to the cutting plane and which passes through the center. The poles of the circle are the two points at which the axis pierces the surface of the sphere. Fig. 2.1 illustrates the above terms.

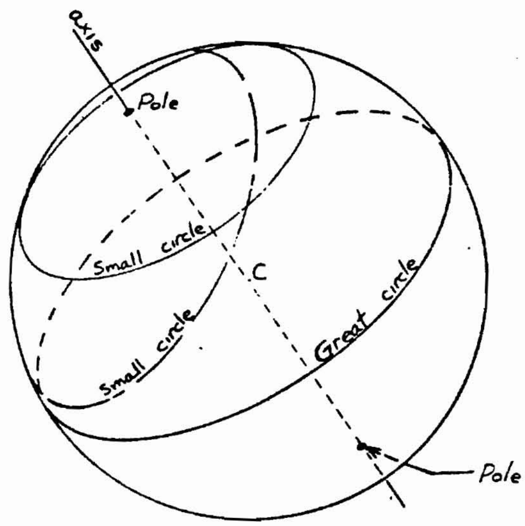
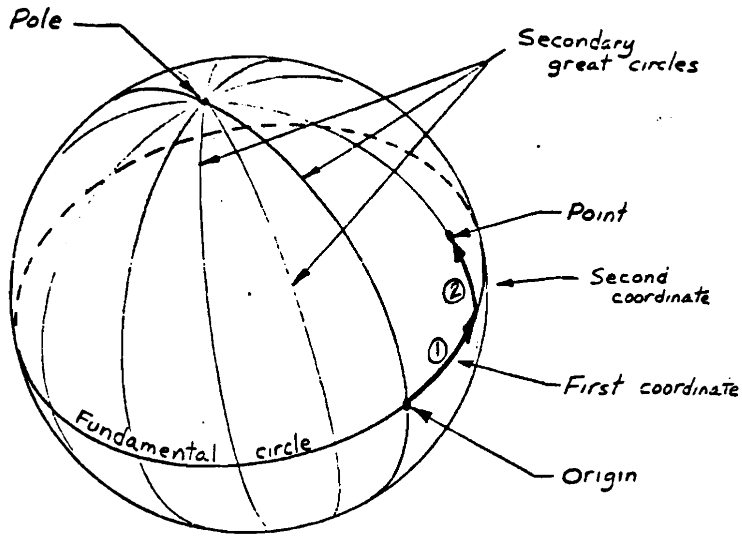


Fig. 2.1 Terms used in spherical geometry.

2.2 Points and Circles of Reference. To determine the position of a point on the surface of a sphere we imagine circles and points of reference as follows (see also Fig. 2.2):

- the fundamental circle, an arbitrary great circle of the sphere.
- the poles of this fundamental circle.
- a family of secondary great circles all of which pass through the poles and are therefore perpendicular to the fundamental circle.
- the origin, an arbitrary point on the fundamental circle.

Two coordinates are necessary and sufficient to locate a point on the surface of a sphere. One coordinate of the point is measured from the origin along the fundamental circle to its intersection with the secondary circle passing through the point; the other is measured along this secondary circle, from the intersection to the point. Observe that a complete definition of a coordinate of a point must include four things: (1) the circle on which it is



Must have:
- fundamental circle
- poles.
- secondary great circles
- origin

2 coordinates

Fig. 2.2 Spherical coordinate system.

measured, (2) the initial point on that circle, (3) the direction of the measurement, and (4) the terminal point.

By assuming different spheres, fundamental circles, and origins we obtain different systems of coordinates. There are five systems which we will use in our work: the geographic, horizon, dependent equatorial, independent equatorial, and ecliptic systems; these are described in detail in the following sections. Only the first of these, the geographic, is a terrestrial system; the other four are celestial.

2.3 The Geographic System. In this system, illustrated in Fig. 2.3, the reference sphere is the earth, which to a first approximation may be considered as a true sphere. The fundamental circle is the earth's equator, and the axis is the rotational axis of the earth; the poles are the earth's geographic north and south poles. The secondary great circles are called meridians of longitude, or simply meridians. The origin is the point of intersection of the equator and the meridian which passes through a specific point in the Royal Observatory in Greenwich, England.

The longitude of a point is the angular distance measured from the Greenwich meridian 2 either eastward or westward 3* along the equator 1 to the meridian of the point 4. It is also seen to be the angle at the pole from the Greenwich meridian to the meridian of the point. The symbol for longitude is the Greek letter λ.

For the sake of systematic procedure and unique definition, it would be better to define the sense in which longitude is measured as always eastward (or else always westward) from Greenwich, counting from 0° to 360°; traditionally, however, the practice is to measure in either direction as

* The numbers appearing below the underlining refer to the corresponding numbers in section 2.2.

convenient, so as to keep the numerical value to 180° or less, and to specify, by adding either the letter E or W, the direction of measurement from the Greenwich meridian. For example, the longitude of a particular place would be expressed in the traditional manner as 145°19'48"E.

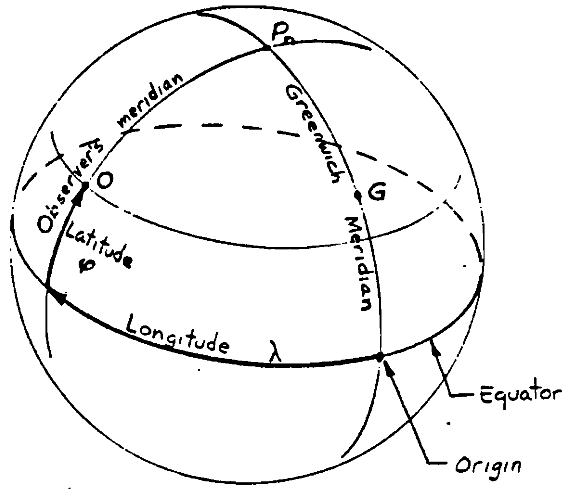


Fig. 2.3 **The geographic system of coordinates.**

The latitude φ of a point on the surface of the earth, here considered to be truly spherical, is the angular distance from the equator [*2] measured northward or southward [3] along the meridian [1] of the point to the point [4]. The sense of measurement is denoted either by adding the letter N or S after the numerical value, or by preceding the numerical value with an algebraic sign, + for north and - for south latitude. For example, the latitude of a certain place might be expressed either as 17° 12' 33"S or as -17° 12' 33".

The definition of latitude as given in the paragraph above is a simplified one based on the assumption of a spherical, non-rotating, homogeneous earth. Depending upon the accuracy required in a particular situation, it may become necessary to recognize the departure of the actual earth from these idealized conditions, and to distinguish between three different kinds of latitude, as described on the following pages.

* The numbers appearing below the underlining refer to the corresponding numbers in section 2.2.

2.3.1 The earth-ellipsoid. The actual shape of the earth's surface is extremely irregular when the various physical features such as mountains and valleys are taken into account. In geodesy, the mean free surface of the oceans, being a surface of equilibrium, is used as the datum to which the heights of various points are referred. This surface, including its imaginary extension under the continents (assuming the continental material to be porous so that the ocean can percolate through it) , is called the geoid. Since the geoid is not a simple mathematical shape, its use for the calculation of geometric positions is unwieldy, so that the usual practice is to refer measurements to an exact ellipsoid of revolution whose size and shape are chosen to coincide as nearly as possible with the geoid. Two parameters are necessary and sufficient to define a rotational ellipsoid; the ones commonly specified are:

- the size parameter a_e, which fixes the equatorial radius, also called the semi-major axis, of the ellipsoid.

- the shape parameter f, called the flattening, which fixes the ellipticity of the cross-section. The flattening f is related to the eccentricity e by the formula

$$(1-f)^2 = 1-e^2 .$$

Over the past several decades, a number of reference ellipsoids have been proposed by different researchers; some of these are given in Table 2.1 . In all cases, the departure of the ellipsoid from a sphere is not large. The ellipsoid to be used in all work involving data from the Astronomical Almanac is the one adopted by the International Astronomical Union (1976). Fig. 2.4 shows, to scale, the relation of the IAU-1976 ellipsoid to a sphere which has the same equatorial radius.

Table 2.1 Reference Ellipsoids

Name	Year	a_e	1/f
Bessel	1841	6,377,397 meters	299.15
Clarke	1866	6,378,206	294.98
Hayford	1909	388	297.0
Heiskanen	1926	397	297.0
Jeffreys	1948	099	297.10
Hough	1956	260	297.0
Fischer	1960	155	298.3
Kaula (IAU-1964)	1964	160	298.25
Veis	1964	169	298.25
SPACETRACK	1968	145	298.25
WGS-72	1972	135	298.26
IAU-1976	1976	140	298.257

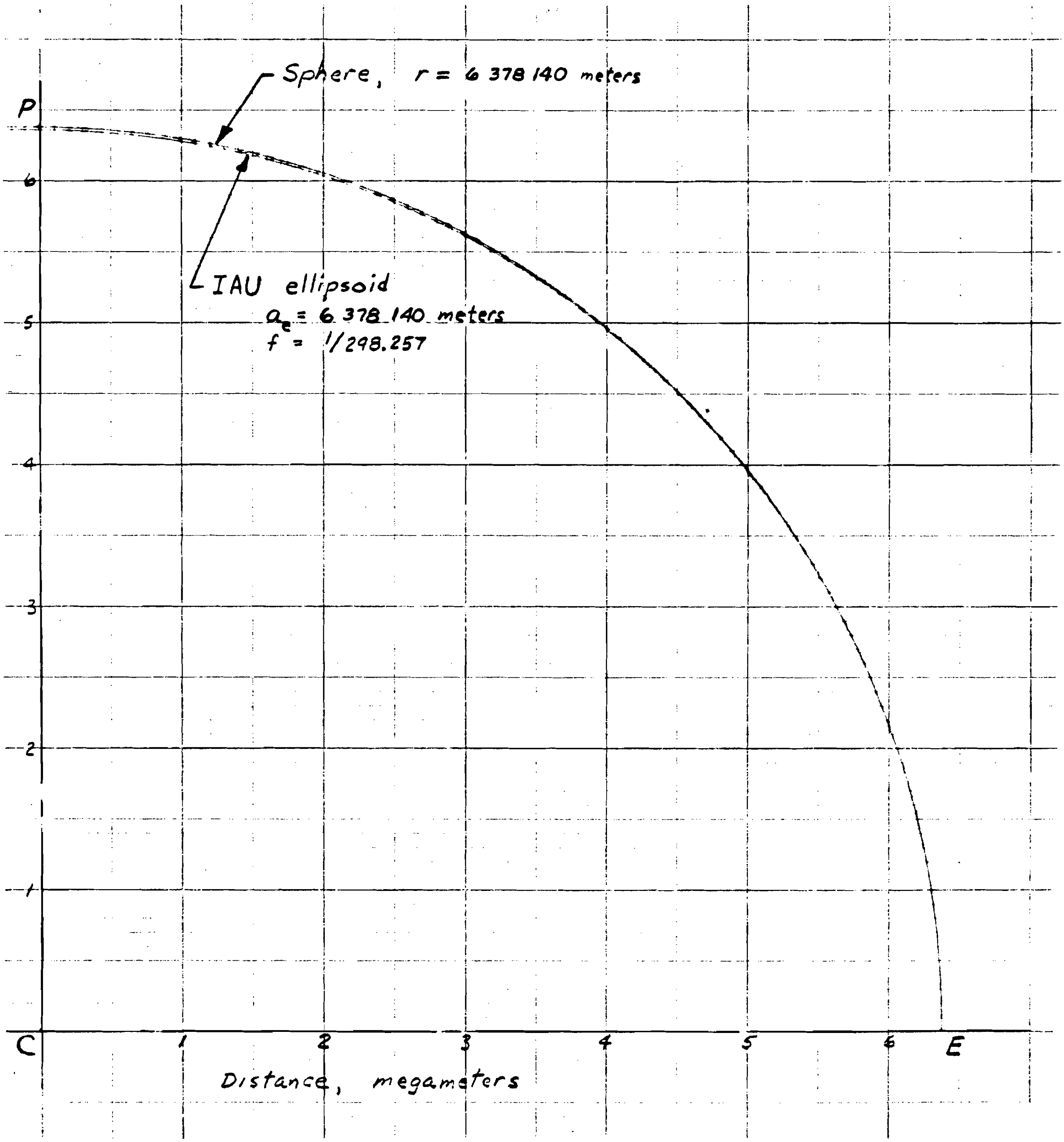


Fig. 2.4 Relation between the IAU Ellipsoid and a true sphere

2.3.2 Geodetic and geocentric latitudes. When the earth is considered as an ellipsoid, rather than as a sphere, it becomes necessary to distinguish between two different kinds of latitude, as defined below.

Fig. 2.5 represents a portion of a meridional cross-section of the earth-ellipsoid with C as the center, CP the polar axis (axis of rotation), and CQ the plane of the equator. The geocentric latitude, denoted by φ', is the acute angle formed at the center of the earth between the equatorial plane and the line joining the center and the observer at O. This definition holds even if the observer is above or below the local terrain.

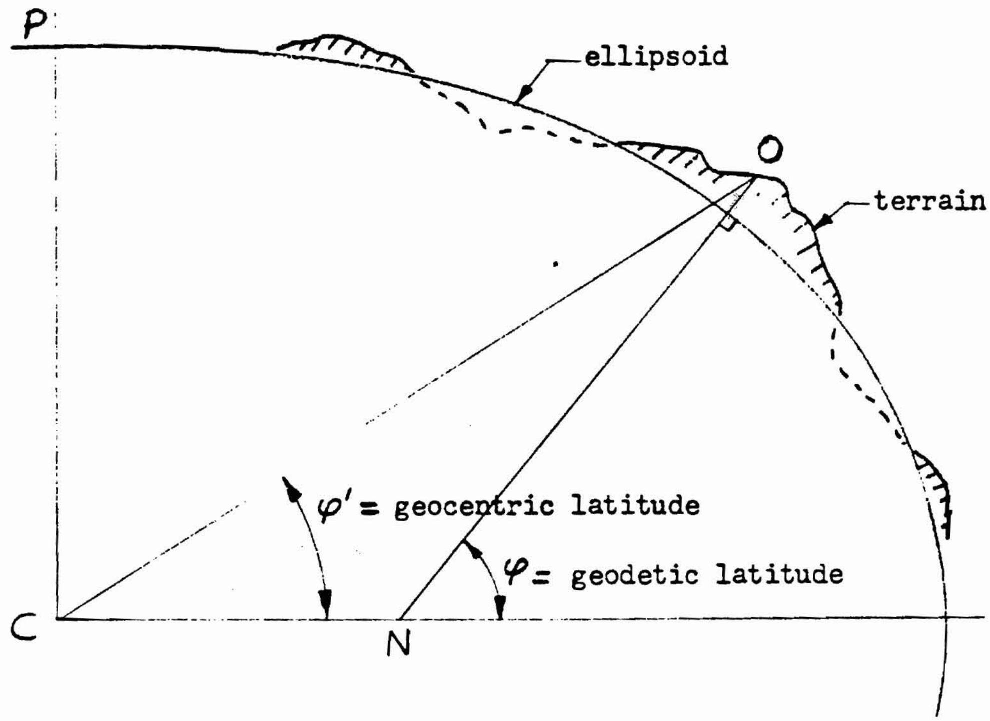


Fig. 2.5 Geocentric and Geodetic Latitudes.

Now let a line be drawn from O so as to cut the ellipse at a right angle; extend this line to intersect the equatorial plane at the point N. The line ON is called the geodetic vertical of O. The geodetic latitude, denoted by φ , is the acute angle formed at the equator between the equatorial plane and the geodetic vertical of the observer. As in the case of geocentric latitude, this definition holds even if the observer is above or below the local terrain.

2.3.3 Relation between geodetic and geocentric latitudes.

Case 1: Geodetic coordinates given, geocentric coordinates required.

Consider the meridional cross-section of the earth-ellipsoid as shown in Fig. 2.6, in which C is the center, CQ represents the plane of the equator, and CP the polar axis. The semimajor axis of the elliptical cross-section is denoted by a and the semiminor axis by b . Point O represents an observer at a geocentric radius r_0 = CO and a geocentric latitude φ' . Point O lies at a geodetic height H_0 = OO" above the ellipsoid, as measured along the geodetic vertical ON, which is normal to the ellipsoid at the geodetic subpoint O". The Cartesian coordinates of O are x_0, y_0 and those of O" are x" , y" .

From ellipse geometry, we know that

$$e = \text{eccentricity} = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$1 - e^2 = (1 - f)^2 = \left(\frac{b}{a}\right)^2 \qquad (2.3\text{-}1)$$

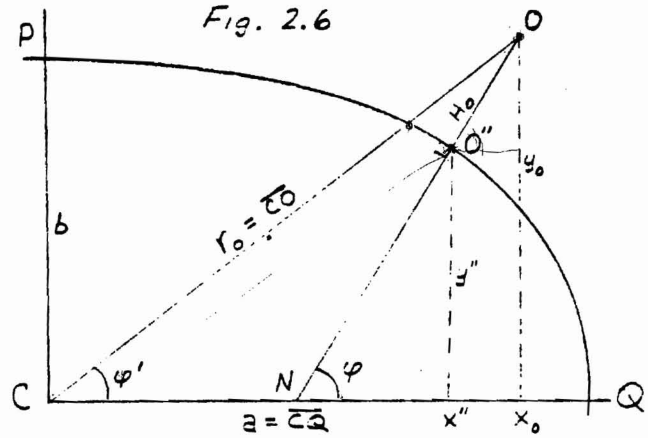


Fig. 2.6

The equation of the ellipse is

$$b^2 x^2 + a^2 y^2 = a^2 b^2$$

From eq. 2.3-1,

$$b^2 = a^2 (1 - e^2)$$

so that we may write the equation of the ellipse as

$$a^2 (1 - e^2)x^2 + a^2 y^2 = a^2 \cdot a^2 (1 - e^2)$$

$$(1 - e^2)x^2 + y^2 = a^2 (1 - e^2) \qquad (2.3\text{-}2)$$

Differentiating this expression,

$$2(1 - e^2)x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(1 - e^2)x}{y} \qquad (2.3\text{-}3)$$

The slope of the normal to the ellipse at any point is the negative reciprocal,

$$\frac{dx}{dy} = \frac{y}{(1 - e^2)x} \qquad (2.3\text{-}4)$$

At the geodetic subpoint O", the slope of the normal is equal to the tangent of the geodetic latitude, that is,

$$\frac{dx}{dy} = \frac{y''}{(1 - e^2)x''} = \tan \varphi \qquad (2.3\text{-}5)$$

$$y'' = x''(1 - e^2)\tan \varphi \qquad (2.3\text{-}6)$$

$$(y'')^2 = (x'')^2 (1 - e^2)^2 \tan^2 \varphi \qquad (2.3\text{-}7)$$

Using eq. 2.3-2, evaluated at point O", in eq. 2.3-7, yields

$$a^2(1 - e^2) - (x")^2(1 - e^2) = (x")^2(1 - e^2)^2 \tan^2\varphi$$

$$a^2 = (x")^2\left[1 + (1 - e^2)\tan^2\varphi\right]$$

$$x" = \frac{a}{\sqrt{1 + (1 - e^2)\tan^2\varphi}} \qquad (2.3\text{-}8)$$

From Fig. 2.6, by inspection,

$$x_o = x" + H_o \cos\varphi \qquad (2.3\text{-}9)$$

$$y_o = y" + H_o \sin\varphi \qquad (2.3\text{-}10)$$

$$r_o = \sqrt{x_o^2 + y_o^2} \qquad (2.3\text{-}11)$$

$$\varphi' = \tan^{-1}\left(\frac{y_o}{x_o}\right) \qquad (2.3\text{-}12)$$

Therefore the geocentric coordinates φ', r_o are determined from the geodetic coordinates φ, H_o .

When the observer is on the surface of the ellipsoid, $H_o = 0$, and eqs. 2.3-8 through 2.3-12 become simplified as follows:

$$x_o = x" = \frac{a}{\sqrt{1 + (1 - e^2)\tan^2\varphi}} \qquad (2.3\text{-}13)$$

$$y_o = y" = x"(1 - e^2)\tan\varphi \qquad (2.3\text{-}14)$$

Using (13) and (14) in (12) gives, after a little rearrangement,

$$\tan\varphi' = (1 - e^2)\tan\varphi \qquad \text{equ: } (2\text{-}1) \qquad (2.3\text{-}15)$$

(The Almanac gives the relation between geocentric and geodetic latitudes in the form of an alternating series which may be obtained from the rigorous expression above by appropriate substitutions and expansions, retaining only the first few terms of the resultant series.)

Using (13) and (14), and by inspection of Fig. 2.6, the expression for the geocentric radius may be put in the form

$$r_o = a\sqrt{\frac{1 + (1 - e^2)^2\tan^2\varphi}{1 + (1 - e^2)\tan^2\varphi}} \qquad (2.3\text{-}16)$$

(Again, the Almanac gives the geocentric radius in the form of an alternating series, obtainable from the rigorous expression above.)

The geocentric radius in this case may be found by substituting the above expression for the geodetic latitude into eq. 2.3-16, to yield, after some rearrangement and reduction, the result

$$r_o = a\sqrt{\frac{(1 - e^2)}{1 - e^2\cos^2\varphi'}} \qquad (2.3\text{-}17)$$

Using either eq. 2.3-15 or the equivalent series formula from the Almanac, Table 2.2 may be constructed; it is valid only for points on the surface of the ellipsoid. The table shows that the geocentric latitude is always less than the geodetic by an amount which varies from 0° at the equator and the poles to about 0°11'33" in geodetic latitude 45° N and S.

Table 2.2 Comparison of Geocentric and Geodetic Latitudes for an Observer on the Surface of the Ellipsoid

φ Geodetic Latitude	Difference *	φ' Geocentric Latitude
0°	0' 0"	0° 0' 0"
5	- 2 0	4 58 0
10	- 3 56	9 56 4
15	- 5 45	14 54 15
20	- 7 24	19 52 36
25	- 8 50	24 51 10
30	- 9 59	29 50 1
35	-10 50	34 49 10
40	-11 22	39 48 38
45	-11 33	44 48 27
50	-11 23	49 48 37
55	-10 52	54 49 8
60	-10 1	59 49 59
65	- 8 52	64 51 8
70	- 7 26	69 52 34
75	- 5 47	74 54 13
80	- 3 58	79 56 2
85	- 2 1	84 57 59
90	0 0	90 0 0

*The values of $\varphi' - \varphi$ shown in Table 2.2 are rounded to the nearest arcsecond, for convenience; more precisely, the maximum difference

$$(\varphi' - \varphi)_{max} = -11'32''.744 \tag{2.3-18}$$

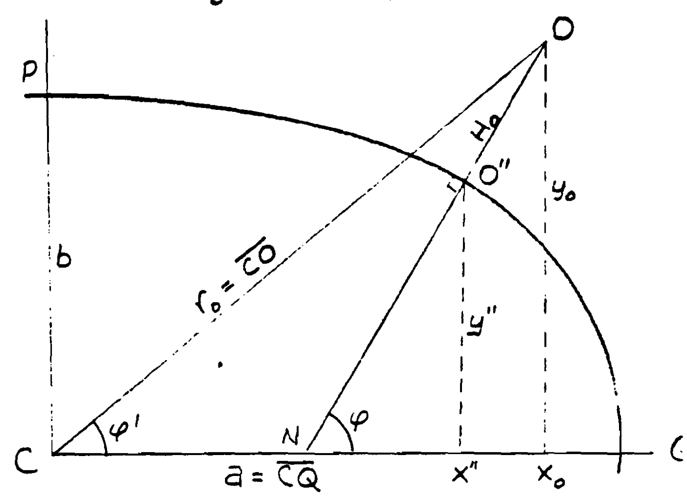
occurs at $\varphi = 45°05'16''.355$ ($\varphi' = 44°54'13''.611$).

Relation between geodetic and geocentric latitudes (continued).

Case 2: Geocentric coordinates given, geodetic coordinates required.

Again referring to Fig. 2.6, repeated here for convenience, let the geocentric coordinates φ', r_o be given, and let it be desired to find the corresponding geodetic coordinates φ and H_o .

Fig. 2.6 (repeated)



The equation of the geodetic normal ON is

$$y'' = y_o + (x'' - x_o) \cdot \frac{y''}{(1 - e^2)\, x''} \qquad (2.3\text{-}19)$$

using the slope expression from eq. 2.3-4,

$$y''\left[1 - \frac{x'' - x_o}{x''(1 - e^2)}\right] = y_o$$

$$y'' = \frac{y_o (1 - e^2) x''}{x_o - e^2 x''} \qquad (2.3\text{-}20)$$

From eq. 2.3-2,

$$y''^2 = a^2(1 - e^2) - x''^2(1 - e^2)$$

$$y''^2 = (a^2 - x''^2)(1 - e^2) \qquad (2.3\text{-}21)$$

Using (20) in (21) gives

$$\frac{y_o^2 (1 - e^2) x''^2}{(x_o - e^2 x'')^2} = (a^2 - x''^2)(1 - e^2)$$

which leads, after a little reduction, to a polynomial in x'',

$$e^4 x''^4 - 2e^2 x_o x''^3 + \left[x_o^2 + (1 - e^2) y_o^2 - e^4 a^2\right] x''^2 + 2e^2 a^2 x_o x'' - a^2 x_o^2 = 0 \qquad (2.3\text{-}22)$$

Having found x'' from (22), y'' is then given by (21), put in the form

$$y'' = \sqrt{(a^2 - x''^2)(1 - e^2)} \qquad (2.3\text{-}23)$$

The geodetic latitude may be found from eq. 2.3-5, repeated here,

$$\tan \varphi = \frac{y''}{(1 - e^2)\, x''} \qquad (2.3\text{-}5)$$

By inspection of the figure, the geodetic height is

$$H_o = \sqrt{(x_o - x'')^2 + (y_o - y'')^2} \qquad (2.3\text{-}24)$$

Therefore the geodetic coordinates φ, H_o are determined from the geocentric coordinates φ', r_o .

When the observer is on the surface of the ellipsoid, $H_o = 0$, and the geodetic latitude is given immediately by eq. 2.3-15, with slight rearrangement, as

$$\tan \varphi = \frac{\tan \varphi'}{(1 - e^2)} \qquad (2.3\text{-}25)$$

2.3.4 Astronomic latitude φ_a. Due to the fact that the earth is not a homogeneous mass, the direction of the gravitational attraction at a particular point is not, in general, directed along the normal to the surface at that same point. Further, the rotation of the earth contributes a centrifugal effect to a plumb bob or level bubble associated with an instrument; this effect is largely confined to the plane of the meridian. The net result is that there exists a local astronomic or gravity vertical which does not coincide with the geodetic vertical (normal to the ellipsoid). As shown in Fig. 2.7, the acute angle between the equatorial plane and the projection of the astronomic vertical into the plane of the meridian is called the astronomic latitude, denoted by φ_a. Fig. 2.7 illustrates the three kinds of latitude: geocentric, geodetic, and astronomic.

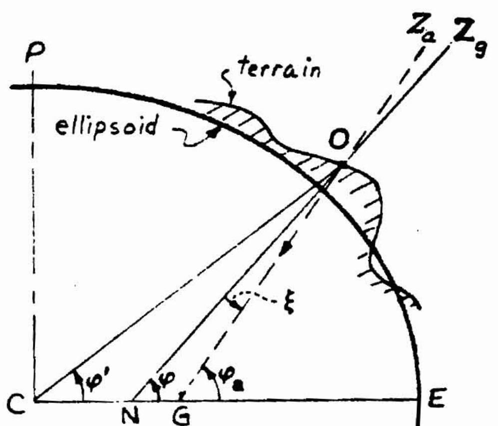


Fig. 2.7 Geocentric, geodetic, and astronomic latitudes: The direction of the earth's gravitational attraction, including centrifugal effects, is along the line OG; the angle EGO is the astronomic latitude of O.

The total angle between the geodetic and astronomic verticals is called the deflection of the vertical. This total angle may be resolved into two components, as follows:

- the meridional component ξ, called the deflection in latitude, considered positive from the geodetic zenith Z_g toward the north celestial pole P; in this case, the plumb bob hangs to the south of the geodetic vertical.
- the prime vertical component η, considered positive from the geodetic zenith towards the east point; in this case, the plumb bob hangs to the west of the geodetic vertical.

The actual values of the deflections are so small that the relations connecting them with the astronomic and geodetic latitudes and longitudes may, with sufficient accuracy, be written as

$$\varphi_a - \varphi = \xi \quad (2\text{-}3)$$

$$(\lambda_a - \lambda)\cos\varphi_a = \eta \quad (2\text{-}4)$$

Differences of 5" between the astronomic and geodetic coordinates are common; differences of 10" are frequent, and differences as large as 20" are not exceptional. In a few rare cases, the deflection may amount to as much as 40", which corresponds to over a kilometer on the surface of the earth.

2.4 The Horizon System. In this system, the celestial sphere is taken as the reference sphere. The plumb line through an observer on the surface of the earth is called the astronomic vertical. This plumb line produced upward from the observer pierces the celestial sphere in a point called the astronomic zenith; the same plumb line produced downward from the observer pierces the celestial sphere in the opposite point, called the astronomic nadir. The plane through the observer and perpendicular to the astronomic vertical intersects the celestial sphere in the astronomic (or celestial) horizon.

(In similar fashion, the line passing through the observer and normal to the ellipsoid is the geodetic vertical, which pierces the celestial sphere in the geodetic zenith and the geodetic nadir. The plane through the observer and perpendicular to the geodetic vertical intersects the celestial sphere in the geodetic horizon. In the discussion which follows, and also in the rest of this text, when the word "zenith" or "nadir" is used, it shall be understood to refer to the astronomic zenith or astronomic nadir, unless preceded by the adjective "geodetic".)

The celestial horizon is the fundamental circle of the system; the zenith and nadir are its poles. Since the surface of still water is always perpendicular to the direction of the plumb line, we may define the celestial horizon for an observer on the surface of the earth as the intersection of the celestial sphere and the plane tangent to the level surface at that point. The visible horizon is the line on the celestial sphere where the earth's surface appears to meet the sky. For an observer on land, the visible horizon may be highly irregular; for an observer on the sea, it is the circle where the sea and sky seem to meet. Projected on the celestial sphere, it is a small circle below the celestial horizon and parallel to it at a distance which depends upon the height of the observer's eye above the surface of the water.

Vertical circles are great circles passing through the zenith and nadir; they are secondary great circles of the system. The vertical circle that passes through the celestial poles (see section 1.5) is called the celestial meridian or simply the meridian. The prime vertical is the vertical circle at right angles to the meridian. The two intersections of the celestial meridian with the horizon are known as the north and south points; those of the prime vertical with the horizon, as the east and west points.

elevation = altitude

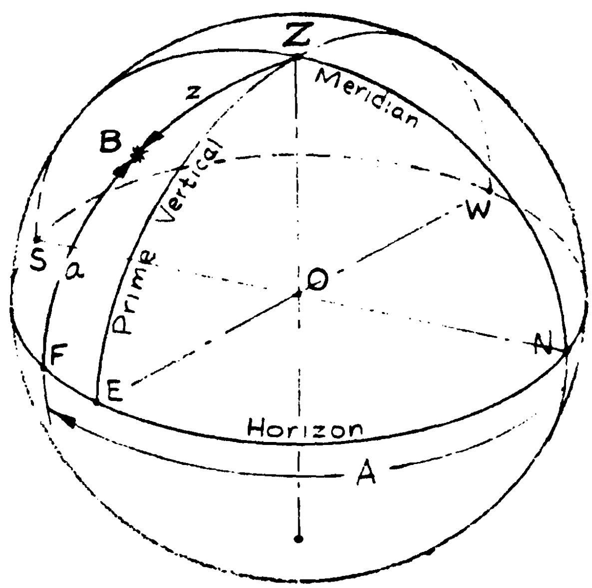


Fig. 2.9 The horizon system of coordinates. The observer is at O, with his zenith at Z. N and S are the north and south points, E and W the east and west points. ZBF is the vertical circle through the body B.

The azimuth A of a heavenly body is the angular distance measured clockwise [3*] (as viewed from the zenith looking toward the nadir) on the horizon [1] from the north point [2] to the foot of the vertical circle through the body [4]. It is also the angle at the zenith from the north branch of the meridian clockwise to the vertical circle through the body.

The altitude a of a heavenly body is the angular distance measured upward [3*] on the vertical circle through the body [1] from the horizon [2] to the body [4].

The zenith distance z is the complement of the altitude a, that is,

$$z = 90° - a. \qquad (2\text{-}8)$$

In some cases, it may be more convenient to work with the zenith distance of a body rather than with its altitude.

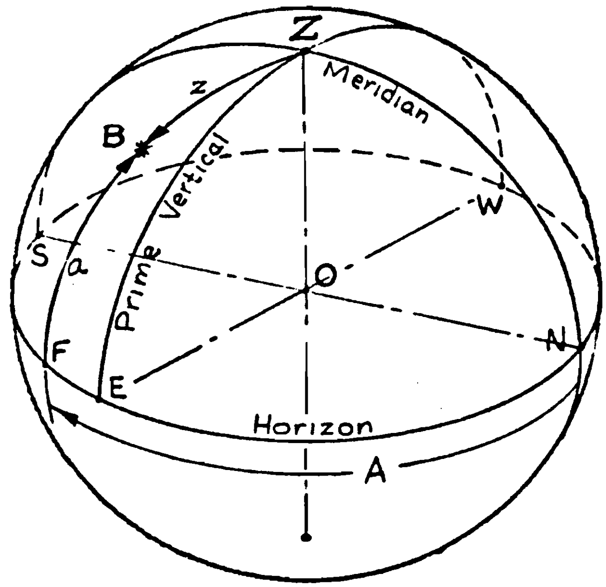


Fig. 2.9 The horizon system of coordinates. The observer is at O, with his zenith at Z. N and S are the north and south points, E and W the east and west points. ZBF is the vertical circle through the body B.

It is important to note the following:

1. The coordinates of a body in the horizon system are not constant; that is, principally on account of the diurnal (daily) motion, the altitude and azimuth of a body such as a star continually change.
2. The horizon system is local; that is, the altitude and azimuth of any body at a given instant are different for two observers situated at different places. Just as soon as an observer changes his position, his zenith changes; hence, also, his horizon and usually his meridian change.

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* The numbers appearing below the underlining refer to the corresponding numbers in section 2.2 .

2.5 The Dependent Equatorial System. The celestial poles have been defined as the two points of intersection at which the earth's rotation axis pierces the celestial sphere. The intersection of the celestial sphere with the plane through the center of the earth which is perpendicular to the rotation axis is called the celestial equator; it is the great circle in which the earth's equator cuts the celestial sphere, as suggested by Fig. 2.10 . Small circles on the celestial sphere which are parallel to the celestial equator are known as parallels of declination; they are also called the diurnal circles.

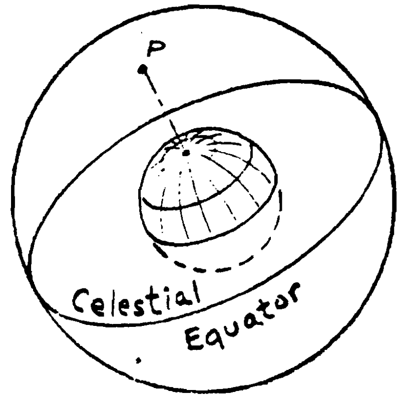


Fig. 2.10 The celestial equator is the intersection of the earth's equator with the celestial sphere. The celestial poles are the points at which the earth's axis produced pierce the celestial sphere.

In the dependent equatorial system, illustrated by Fig. 2.11, the celestial equator is taken as the fundamental circle with the celestial poles as its poles. The secondary great circles are the great circles perpendicular to the equator and are known as hour circles. The half of the circle lying between the poles and containing a heavenly body is referred to as the hour circle of the body. The hour circle through the zenith of an observer is the meridian of the observer. Note that the meridian of the observer is both an hour circle and a vertical circle. The upper branch of the meridian is that half extending from pole to pole which contains the zenith; the other half contains the nadir and is called the lower branch.

The point at which the upper branch of the observer's meridian intersects the celestial equator is referred to as the sigma point Σ .

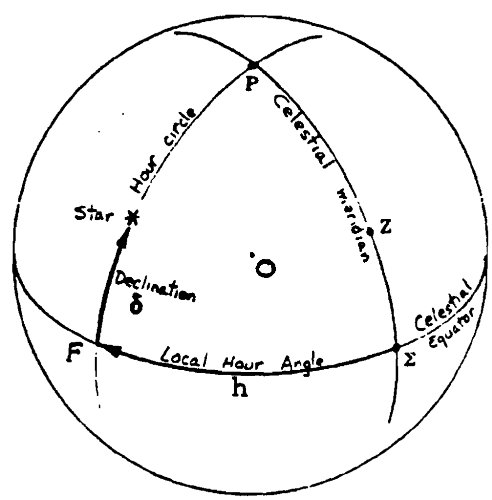


Fig. 2.11 The dependent equatorial system of coordinates. The observer is at O; P is the north celestial pole. The star is located by its local hour angle and declination.

The local hour angle (h, LHA), or simply the hour angle, of a heavenly body is the angular distance measured westward on the equator from the point of
3* 1
intersection of the meridian and equator (Σ point) to the foot of the hour
2
circle through the body; it is also the angle at the pole from the meridian
4
measured westward to the hour circle through the body. The local hour angle is expressed either in degree or in hour units. For example, the local hour angle of the west point may be given either as 90° or as 6^h. The LHA of the east point may likewise be given either as 270° or as 18^h.

The declination δ of a heavenly body is the angular distance measured on the hour circle through the body from the equator to the body. It is
1* 2 4
named north or south according as the body is either north or south of the
3
equator. Alternately, it may be designated as plus when north or minus when south of the equator.

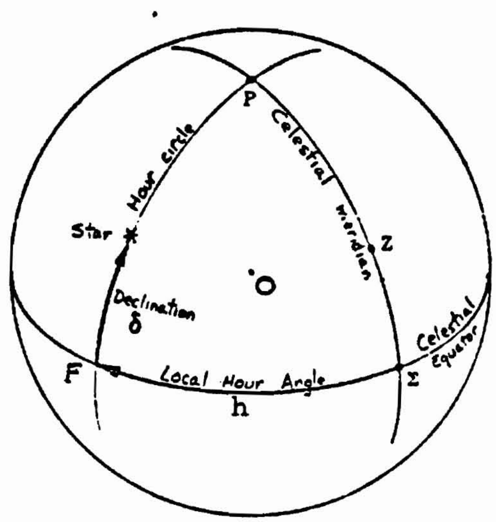


Fig. 2.11 The dependent equatorial system of coordinates. The observer is at O; P is the north celestial pole. The star is located by its local hour angle and declination.

The local hour angle and declination of a heavenly body at a given instant determine its position on the celestial sphere at that instant. It is important to note that the coordinates of a body in this system are not constant; the local hour angle continually changes since it is measured from a point (the Σ point, on the meridian of the observer) which is not carried along in the diurnal rotation of the celestial sphere.

* The numbers appearing below the underlining refer to the corresponding numbers in section 2.2.

2.6 The Independent Equatorial System. To obtain a system in which the coordinates are not affected by the diurnal motion, we assume a point on the celestial sphere that is carried along by the diurnal rotation and that is therefore fixed (as far as possible) with respect to the stars. The point which has been chosen is the point at which the sun crosses the celestial equator from south to north; this point has already been discussed (sec. 1.6) and is called the vernal equinox (it may also be referred to as the ascending node of the apparent orbit of the sun). The symbol ♈ is traditionally used to represent this point, being the symbol used for the sign of Aries, the Ram, in which constellation this point of intersection was located at the time of the original definition. The letter V is also used to denote this point.

The right ascension α of a heavenly body is the angular distance measured eastward [3*] on the equator [1] from the vernal equinox [2] to the foot of the hour circle through the body [4]. Right ascension is usually expressed in time units (hours, minutes, and seconds).

Fig. 2.12 shows the celestial sphere with the equator and ecliptic intersecting at ♈, the vernal equinox. The hour circle through the vernal equinox is called the equinoctial colure. The star shown is located by giving its right ascension and declination; note that the right ascension is measured eastward from the vernal equinox.

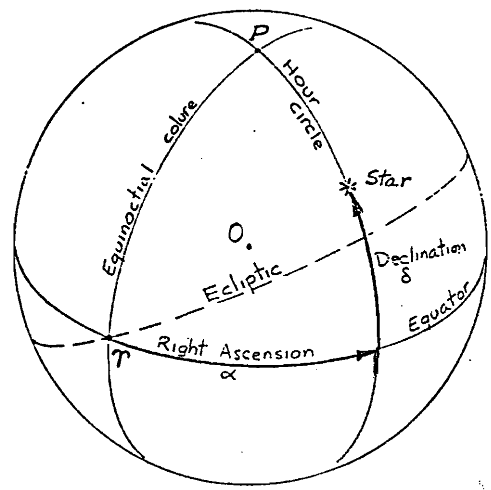


Fig. 2.12 The independent equatorial system. The star shown is located by giving its right ascension and declination.

The right ascension and declination of a star remain practically constant for years, hence they are well suited for defining the position of a star on the celestial sphere. The mean place of a star is the right ascension and declination of the star as seen by an observer located at the center of the sun, for an epoch (instant of time) near the middle of the current year. Mean places for 1,482 stars are given in the Almanac, in section H.

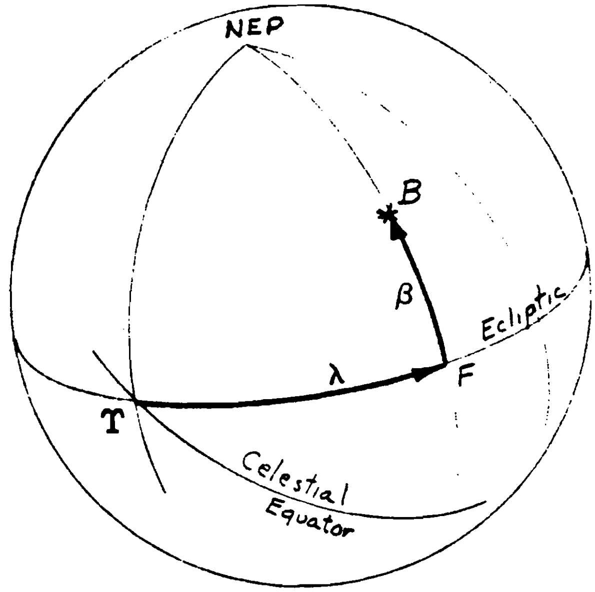
* The numbers appearing below the underlining refer to the corresponding numbers in section 2.2.

2.7 The Ecliptic System. In this system, the celestial sphere is the reference sphere and the ecliptic is taken as the fundamental circle. The two points on the celestial sphere which are 90° from the ecliptic are called the north and south ecliptic poles. The secondary circles through the poles are called ecliptic meridians. The origin of coordinates is the vernal equinox ♈ , which is the ascending node of the ecliptic upon the celestial equator. The two spherical coordinates required for the unique specification of position in this system are described below and are illustrated in Fig. 2.13 .

The ecliptic longitude λ of a heavenly body is the angular distance measured eastward[3*] on the ecliptic[1] from the vernal equinox[2] to the foot of the ecliptic meridian[4] through the body. Ecliptic longitudes are usually expressed in degrees, minutes, and seconds of arc (or in decimal degrees). The choice of the symbol λ for ecliptic longitude is unfortunate, since the same symbol is also used to denote geographic longitudes on the earth; one must depend upon the context to make it clear as to which kind of longitude is meant in a particular situation.

The ecliptic latitude β of a heavenly body is the angular distance from the ecliptic[2*] to the body[4] measured along the ecliptic meridian[1] through the body. It is named north or south[3] according as the body is either north or south of the ecliptic. Alternately, it may be designated as positive when north or negative when south of the ecliptic. In either case, ecliptic latitudes are usually expressed in degrees, minutes, and seconds of arc (or in decimal degrees).

Fig. 2.13 The Ecliptic System of Coordinates.



The ecliptic longitude and latitude of stars remain practically constant for years; this is not so in the case of either the sun or the moon. The ecliptic latitude of the sun does remain very nearly zero, within about 1" of arc, but its ecliptic longitude increases at about 1° per day. The ecliptic longitude of the moon increases at about 13° per day; its ecliptic latitude lies in the range of about ±5°, due to the inclination of the moon's orbit to the ecliptic.

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* The numbers appearing below the underlining refer to the corresponding numbers in section 2.2 .

2.7 Summary of the Coordinate Systems. The one terrestrial and four celestial coordinate syatems which have been described in this chapter are summarized below. The student should clearly understand each system separately before proceeding to the following sections of the chapter which will consider various combinations of the systems and the relations between them.

Table 2.3 Coordinate Systems Summary

	Terrestrial	Celestial			
		Horizon system	Equator system		Ecliptic system
			Dependent	Independent	
Reference sphere	Earth	Celestial sphere	Celestial sphere	Celestial sphere	Celestial sphere
Fundamental circle	Equator	Horizon	Equator	Equator	Ecliptic
Poles	Terrestial poles	Zenith and nadir	Celestial poles	Celestial poles	Ecliptic poles
Secondary great circles	Meridians	Vertical circles	Hour circles	Hour circles	Ecliptic meridians
Names of coordinates	Longitude λ Latitude φ	Azimuth A Altitude a	Hour angle h Declination δ	Right ascension α Declination δ	Ecliptic longitude λ Ecliptic latitude β
Origin	Intersection of meridian of Greenwich with equator	North point	Intersection of meridian with equator (Σ point)	Vernal equinox ♈	Vernal equinox ♈
Positive direction of first coordinate	Westward	Clockwise	Westward	Eastward	Eastward
Positive direction of second coordinate	Northward	Upward	Northward	Northward	Northward

2.8 Altitude of the Celestial North Pole.-- Except for observers located on the earth's equator, only one of the two celestial poles can be seen from any one point on the earth's surface, the other being below the horizon. The one which can be seen, i.e., the one which is above the horizon, is called the elevated pole, the other being the depressed pole. The celestial north pole is always elevated for northern hemisphere observers, but is always depressed for observers in the southern hemisphere. A simple relation between the altitude of the celestial north pole and the latitude, north or south, of the observer may be derived as shown in the following paragraph.

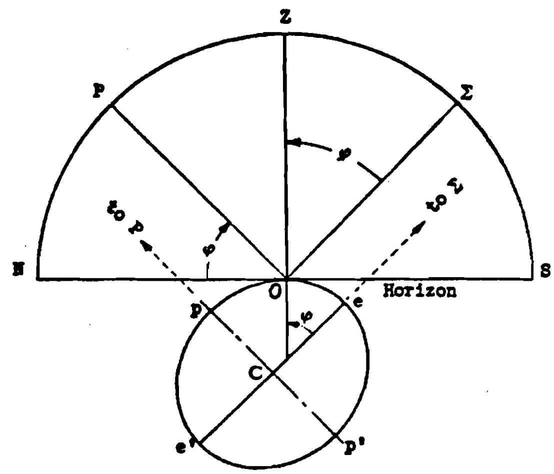


Fig. 2.14 The altitude of the celestial pole P is equal to the astronomic latitude of the observer at O.

The ellipse pep'e' in Fig. 2.14 represents the terrestial meridian of an observer at O. The direction of the plumb line makes the angle φ with the line ee', where e and e' are points on the terrestrial equator. The angle φ has already been defined as the astronomic latitude (section 2.3.4). Since ZO is perpendicular to the horizon NOS and OP is perpendicular to ee', then angle NOP is equal to φ, that is, the altitude of the celestial north pole is equal to the astronomic latitude of the observer. Fig. 2.14 also suggests another definition of astronomic latitude as the zenith distance of the Σ point.

2.9 Meridian Altitude. When a heavenly body crosses the upper branch of the meridian of an observer, its altitude is the greatest and its zenith distance is the least. The passage of a heavenly body across the meridian is known as its transit (referring to the act of crossing) or as its culmination (referring to its greatest altitude). When the body crosses that part of the meridian which is nearer the zenith, it is said to be at upper transit or upper culmination; when it crosses the part farther from the zenith, at lower transit or lower culmination. At a given place there are certain stars that have both upper and lower culmination above the horizon; such stars are known as circumpolar for that place. For example, the stars in Fig. 1.3 are circumpolar at the place for which the figure is drawn.

Many observations of heavenly bodies are made at the meridian. A simple relation between the latitude of the observer, the declination of the body, and its meridian zenith distance (and therefore its meridian altitude) will be given here.

Suppose in Fig. 2.15, that NZS represents the meridian of the observer, and B is a body just crossing the meridian. Then

$\widehat{\Sigma Z}$ = latitude of the observer = φ
$\widehat{ZB}$ = meridian zenith distance of body = z_m
$\widehat{\Sigma B}$ = declination of body = δ

For a body on the upper branch of the meridian, i.e., anywhere on the arc PZS, and if the usual sign convention for declination and latitude is followed (north +, south -), then the meridian zenith distance of the body is given by

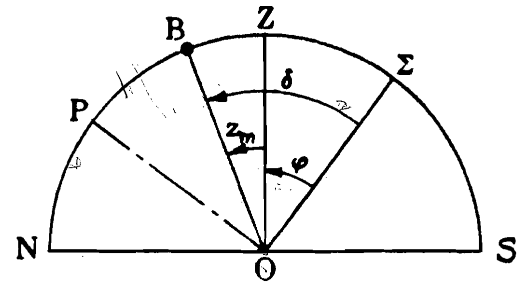


Fig. 2.15 The relation between the latitude of a place, the meridian zenith distance of a body, and its declination.

$$z_m = \delta - \varphi . \tag{2-9}$$

A positive result means that the body is north of the zenith; a negative result means that it is south.

For a body on the lower branch of the meridian, i.e., anywhere on the arc NP, the meridian zenith distance is given by

$$z_m = 180° - \delta - \varphi . \tag{2-10}$$

The meridian altitude is given by

$$a_m = 90° - |z_m| . \tag{2-11}$$

2.10 Orienting the Coordinate Systems on the Celestial Sphere. It is of great importance that the student should master the systems of coordinates and be able to imagine readily the circles and points of reference on the celestial sphere. One way to help the learning process is to go outside on a clear night, preferably when the moon is not in the sky, and look at the heavens themselves.

Directly overhead is the zenith; the north celestial pole will be within 1° of Polaris, the North Star. Remember that the altitude of the pole, and therefore Polaris (approximately), is equal to the latitude of the observer; also, the pole lies very nearly on a line drawn through the pointer stars of the Big Dipper (Fig. 1.8) and about 30° from them. The distance between the pointers is about 5°. There is no other bright star besides Polaris in that region of the sky; Polaris itself is about second magnitude.

The great circle through the zenith and (approximately) Polaris is the meridian. The north point is nearly underneath Polaris; the south point is opposite on the horizon. 90° on either side of these points are the east and west points, also on the horizon. (As is usual in astronomical work, the word horizon means the celestial horizon, unless otherwise indicated.) To locate the Σ point, face south and look up at an angle of 90° - the latitude. A great circle slanting across the sky joining the east, Σ, and west points will determine the celestial equator.

There is, unfortunately, no bright star near the vernal equinox; imagine it as being south of the Great Square of Pegasus, on the equator. The autumnal equinox is about halfway between the bright stars Spica and Regulus.

Having thus located the circles and points of reference, the coordinates (A, a), (h, δ), and (α, δ) of a heavenly body may be estimated.

2.11 Relations Among the Coordinate Systems. A useful concept to aid in visualizing the relations among the four celestial coordinate systems is to imagine that the celestial sphere consists of two thin concentric spherical shells, one within the other. The outer one, the true celestial sphere, carries upon its surface the ecliptic, equinoxes, poles, equator, hour circles, declination circles, the stars, sun, moon, and the planets; on the inner sphere are the zenith, horizon, vertical circles, poles, equator, hour circles and the meridian. The diurnal rotation of the earth from west to east carries the inner sphere with it while the outer (celestial) sphere remains motionless; or, considering the apparent motion, the inner sphere is stationary while the outer sphere rotates from east to west once each day.

Equations giving exact relationships between the coordinate systems will be developed in later portions of this book. Approximate values may be obtained by making an accurate drawing on a convenient spherical surface or on paper and estimating or scaling the desired values from the drawing.

2.12 Relation Between Local Hour Angle and Right Ascension. The two concentric spheres discussed in the preceding article are shown in Fig. 2.16, with the equator on the outer sphere being graduated in hours, minutes, and seconds of right ascension, zero being at the vernal equinox and the numbers increasing toward the east. The equator of the inner sphere is graduated for hour angles, the zero being at the Σ point and the numbers increasing toward the west. As the outer sphere turns, the hour marks on the right ascension scale will pass the meridian in order of the numbers. The number opposite the meridian at any instant shows how far the sphere has turned since the vernal equinox was on the meridian. If we read the LHA scale opposite the equinox, we obtain exactly the same number of hours. This number of hours (or angle) may be considered as either the right ascension of the meridian or as the LHA of the vernal equinox.

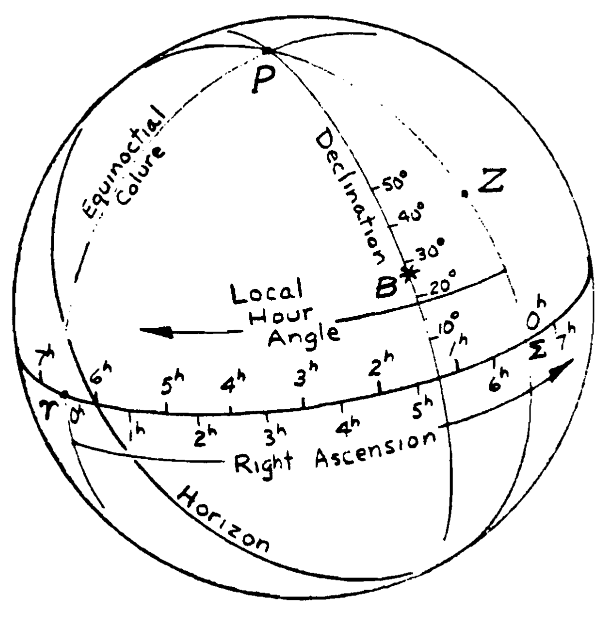


Fig. 2.16 Right ascension and hour angle.

In Fig. 2.17, the star at B has a LHA of ΣF and a right ascension of ♈F; the sum of these two angles is Σ♈, the LHA of the equinox. The same relation holds for all positions of B, so that the general relation between these coordinates is given by

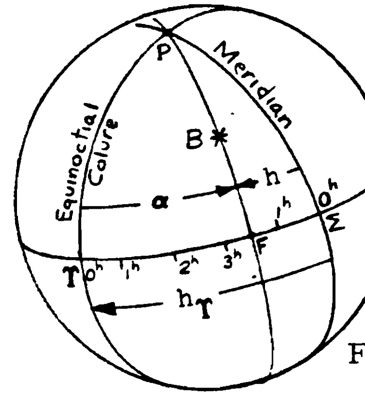


Fig. 2.17

$$h_♈ = h + \alpha \qquad (2\text{-}12)$$

2.13 <u>Changes of the Coordinates of the Sun.</u> (a) Change of right ascension and declination during the year.- On about March 21 the sun crosses the equator from south to north (Fig.14) at the vernal equinox, hence its coordinates are then $\alpha = 0^h$, $\delta = 0°$. Since the sun moves eastward on the ecliptic at about 1° per day, both its right ascension and declination increase so that on about June 21 the sun reaches the summer solstice; its coordinates are then $\alpha = 6^h$, $\delta = +23\frac{1}{2}°$ (approximately). After this date, the declination of the sun begins to decrease while its right ascension continues to increase. The autumnal equinox is reached on about Sept. 22, the sun then having $\alpha = 12^h$ and $\delta = 0°$; thereafter the declination continues to decrease, becoming $-23\frac{1}{2}°$ (approximately) at the winter solstice, on about Dec. 22. The right ascension at this point is 18^h. During the next three months, the sun moves northerly, returning to the vernal equinox to complete the yearly cycle. The right ascension and declination of the sun for each day of the current year are given in the Ephemeris.

(b) Change of LHA and declination during the day.- From the preceding paragraph, we see that the declination of the sun changes only a small amount each day, the most rapid daily change taking place at the equinoxes; even then, the daily change is only about one-third of a degree. The LHA, however, increases at the rate of about 15° per hour, from 0° at the local meridian to 360° the next day when it returns to the same meridian.

(c) Change of azimuth and altitude during the day.- To trace the changes of these coordinates, the latitude of the observer and the date must be known. For example, for an observer at the equator on March 21, the sun rises due east, i.e., its azimuth is 90° and its altitude is 0°. The altitude increases to 90° when the sun reaches the zenith, while the azimuth remains the same. In this case, sunset occurs at the west point ($A = 270°$, $a = 0°$).

For an observer at the north terrestrial pole on March 21, the sun traces approximately the horizon throughout the 24 hours, while for the same observer on June 21, the altitude of the sun remains approximately equal to $23\frac{1}{2}°$ all day long.

For observers at intermediate values of north latitude on June 21, the azimuth of the sun at sunrise will be between 0° and 90°, its maximum altitude will be reached at the meridian, and its azimuth at sunset will be between 270° and 360°.

Fig. 2.18 shows the celestial sphere at about June 1 of the current year; the sun is nearly at the summer solstice SS; the right ascension and declination of the sun, taken from the Ephemeris for the date of interest, enable its position to be plotted. Its altitude and azimuth as seen by an observer whose zenith is at Z may be roughly estimated from the figure; better values can be estimated from an accurate drawing on a sphere.

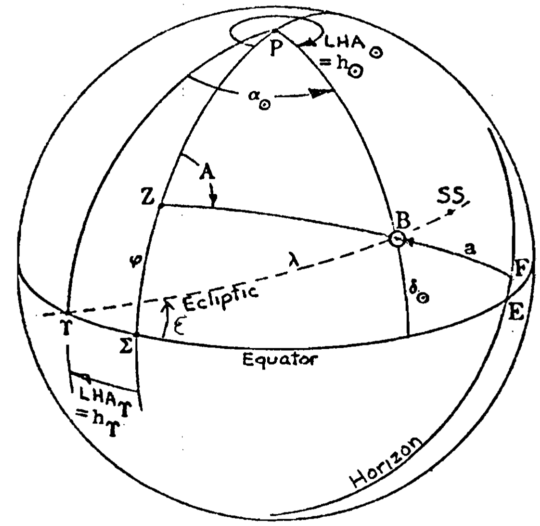


Fig. 2.18 The celestial sphere, showing the sun's location by means of the four celestial coordinate systems.

ε = obliquity of ecliptic

Exercises

2-1. The geodetic latitude of EMOS (East Mesa Optical Site, Sandia National Laboratories, Albuquerque, NM) is 35° 3' 6" N. Using the expression for a degree of latitude from the Almanac, Section K, calculate the distance in meters corresponding to one second of arc at this latitude.

2-2. Same as 2-1, except for one second of longitude.

2-3. Find the geocentric latitude of the site (a) using the formula given in the Almanac, Section K, and (b) using equation 2.3-15 from the text.

2-4. Give the azimuth corresponding to each of the following directions:

(a) SE – 135°	(e) S – 180°
(b) NW – 315°	(f) SW – 225°
(c) E – 90°	(g) W – 270°
(d) NE – 45°	(h) N – 0°

2-5. A star located 15° south of the celestial equator transits (crosses) the meridian of the observer at 19:00 on a certain day; what will be its local hour angle and declination at 22:30 that same day? 15°S 3:30 h (52° 30′)

2-6. Give the right ascension, declination, ecliptic longitude, and ecliptic latitude of the autumnal equinox and of the summer solstice.

2-7. Give the approximate right ascension, declination, ecliptic longitude, and ecliptic latitude of the sun on March 21 and on December 21.

Mar. 20 16h 14m   Sept 23 02h 07m
Jun 21 10h 44m   Dec. 21 22h 08m

2-8. The 'Phenomena' section of the Almanac contains the epochs of the equinoxes and solstices for the current year, given to a precision of one minute of time. By inspection of the Almanac tables of the Sun, verify these four epochs.

2-9. On a suitable star chart, plot the sun's position at 0^h E.T. each day for the dates as specified by the instructor.

2-10. Repeat, but for the moon.

2-11. Find, for latitude φ, the meridian altitude at upper transit of stars of given declination δ, as follows:

	φ	δ		φ	δ
(a)	40°N	20°N	(e)	20°S	5°N
(b)	60°S	10°N	(f)	80°N	2°S
(c)	45°N	15°N	(g)	15°S	60°N
(d)	50°S	35°S	(h)	30°N	89°N

2-12. What is the meridian altitude of Sirius for an observer in latitude 40°N?

2-13. Give the altitude of Dubhe at upper and lower transit for Lowell Observatory.

2-14. The meridian altitude of Regulus above the southern horizon of an observer is 72° 24' 44". What is the observer's latitude?

2-15. Determine whether or not Capella is circumpolar for Sandia Lab observatory.

2-16. What is the approximate meridian altitude of the sun on April 30 for the U.S. Naval Observatory in Washington, D.C.?

2-17. The meridian altitude of the sun above the southern horizon of a ship at sea on May 10 is 20° 17'; find the approximate latitude of the ship.

2-18. Find the azimuth and altitude of the east point for an observer at each of the following latitudes:

(a)	0°N	(d)	60°N
(b)	30°N	(e)	45°S
(c)	45°N	(f)	60°S

2-19. Give the azimuth and altitude of the west point, the north celestial pole, and the Σ point.

2-20. Give the LHA and declination of the east point, the zenith, and the south point. 18h (270°) $\delta = 0°$ $\delta = -90° + \varphi$ (lat)

2-21. Give the right ascension, declination, and LHA of the east point at the instant when the vernal equinox is at the west point.
$\alpha = 12h$ $\delta = 0°$ LHA = 18h (270°)

2-22. Find the right ascension, declination, and altitude of the north point at the instant when the autumnal equinox is at the west point.
$\alpha = 6h$ $\delta =$ ~~90+φ (lat)~~ $90 - \varphi$ $a = 0°$

For the following problems, make a neat sketch on an actual sphere (such as a table tennis ball), and estimate the answers from your sketch.

2-23. An observer at 40°N latitude observes an airplane at 60° azimuth, 30° altitude. The LHA of the vernal equinox at that instant is 6^h. Estimate the LHA, r.a., and dec. of the airplane's apparent position on the celestial sphere.

2-24. Same, but the observer is at 40°S lat.

2-25. A rocket is launched from a ship at 120°W longitude, 20°N latitude, and shortly thereafter reaches an apparent position of 3^h LHA, 10°N declination as seen from the ship. Estimate the azimuth and altitude of the rocket at that time.

2-26. Same, but the ship is at 20°S lat.

2-27. At a certain time, the star Capella is observed at an azimuth of 300°; its LHA at that instant is 2^h. Find:

(a) latitude of the observer
(b) LHA of the vernal equinox.

2-28. Find the LHA, azimuth, and altitude of the sun at sunrise on May 15 of the current year as seen from latitude 35°N.

2-29. Same, but for latitude 35°S.

CHAPTER 3

SPHERICAL TRIGONOMETRY AND THE ASTRONOMICAL TRIANGLE

3.1 <u>The Spherical Triangle.</u> In many problems of practical astronomy it becomes necessary to transform from one system of coordinates to another; this transformation involves the solution of a spherical triangle. A complete treatment of the subject of spherical trigonometry is beyond the scope of this text, but derivations of a few fundamental formulas will be given in the paragraphs which follow. In addition, several other formulas which may be useful in our work will be presented without proof. For complete derivations and proofs of all formulas, reference may be made to any complete text on the subject of spherical trigonometry.

When any three points A, B, and C on the surface of a sphere are joined by arcs of great circles, eight spherical triangles are formed, some of which may have one or more sides or angles greater than 180°. In this text, however, we shall consider only those triangles in which no side or angle exceeds 180°.

Fig. 3.1 shows an octant of a sphere having a radius of unity, and three points A, B, and C lying in the surface of the sphere, forming the spherical triangle ABC. The following statements are true, by construction:

CP ⊥ plane XOY
CP' ⊥ OB
CM ⊥ OA.

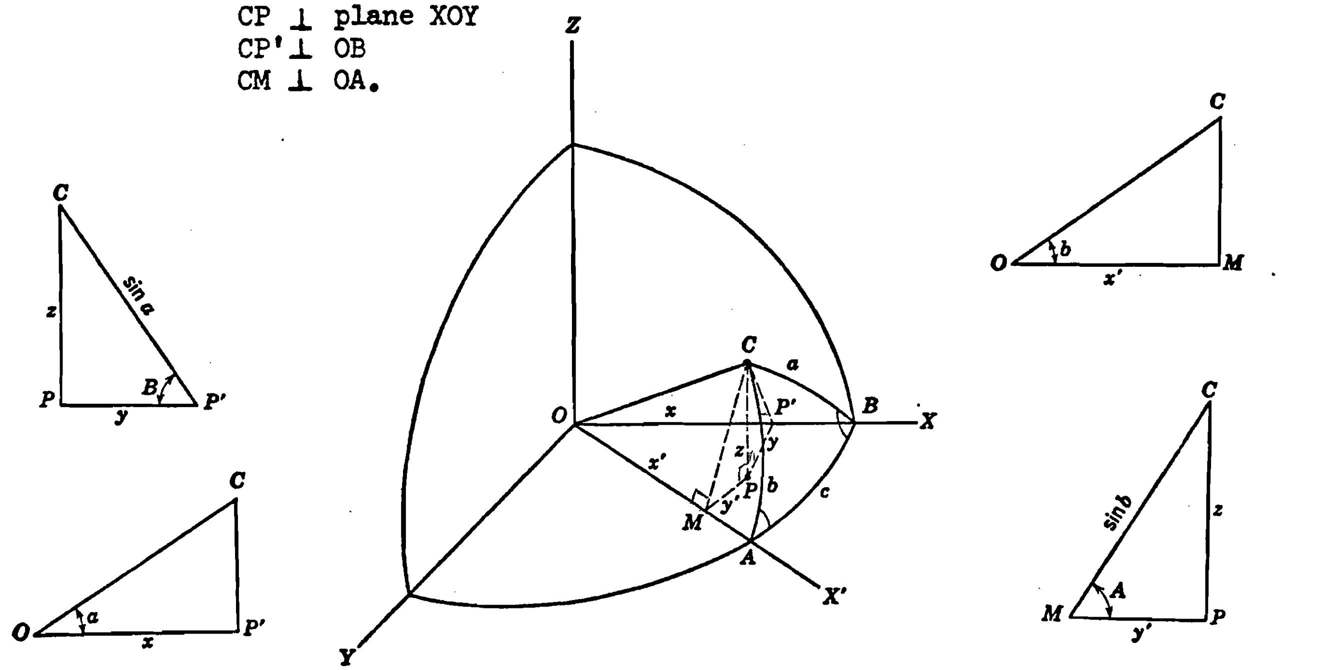


Fig. 3.1 The oblique spherical triangle ABC.

It follows that

$$\left.\begin{aligned} \angle COP' &= a \\ \angle COM &= b \\ \angle P'OM &= c \end{aligned}\right\} \text{ since an arc is equal in measure to its central } \angle.$$

$$\left.\begin{aligned} \angle CP'P &= \angle B \\ \angle CMP &= \angle A \end{aligned}\right\} \text{ by projection.}$$

For radius unity, a study of the small plane triangles involved in the figure shows that

$$x = \cos a \qquad\qquad x' = \cos b$$
$$y = \sin a \cos B \qquad\qquad y' = \sin b \cos A$$
$$z = \sin a \sin B \qquad\qquad z = \sin b \sin A$$

Fig. 3.2 represents the XOY plane of Fig. 3.1, redrawn with a few additional construction lines; from the figure, it is seen that

$$x = x'\cos c + y'\sin c$$
$$y = x'\sin c - y'\cos c$$

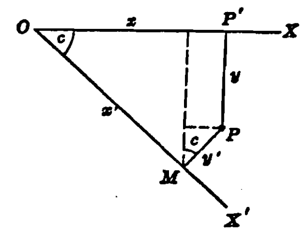


Fig. 3.2

By substitution of the first of these two sets of expressions into the second, we get

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \qquad (3\text{-}1)$$
$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A \qquad (3\text{-}2)$$
$$\sin a \sin B = \sin b \sin A \qquad (3\text{-}3)$$

Corresponding formulae may be written involving angles B and C in the right-hand member of each of the above expressions.

Eq. 3-1 is known as the <u>law of cosines</u>, and may be regarded as the fundamental formula of spherical trigonometry because all others may be derived from it and by means of it all problems in spherical trigonometry may be solved, although not always so conveniently as with other special forms. Eq. 3-2 is called the <u>five-parts formula</u> since it involves five of the six parts of the triangle. Eq. 3-3 is known as the <u>law of sines</u>, which may be written with greater generality as

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \qquad (3\text{-}4)$$

by writing the corresponding forms of Eq. 3-3 and rearranging.

In the preceding derivation, the sides of the spherical triangle were taken as less than 90° for convenience, to permit the construction of the right triangles OCP', OCM, etc. in Fig. 3.1. Reference to a complete text on the subject will show that Eqs. 3-1 through 3-3 also hold for triangles with one or more sides or angles greater than 90° but less than 180°.

By combining and rearranging Eqs. 3-1, 3-2, and 3-3 various other formulas may be obtained; for example, dividing Eq. 3-3 by Eq. 3-2 gives

$$\frac{\sin a \sin B}{\sin a \cos B} = \frac{\sin b \sin A}{\cos b \sin c - \sin b \cos c \cos A}$$

$$\tan B = \frac{\sin A}{\cot b \sin c - \cos c \cos A} \qquad (3\text{-}5)$$

known as the <u>four consecutive parts formula</u>.

3.1.1 Several other formulas which may likewise be derived from Eqs. 3-1, 3-2, and 3-3 are given below, without proof:

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a \qquad (3\text{-}6)$$

$$\tan \frac{A}{2} = \sqrt{\frac{\sin(s-b)\ \sin(s-c)}{\sin s\ \sin(s-a)}} \qquad (3\text{-}7)$$

where $s = \frac{a+b+c}{2}$ = semiperimeter

$$\tan \frac{a}{2} = \sqrt{\frac{-\cos S\ \cos(S-A)}{\cos(S-B)\cos(S-C)}} \qquad (3\text{-}8)$$

where $S = \frac{A+B+C}{2}$ = semigonometer

$$\tan \frac{a+b}{2} = \frac{\cos \frac{A-B}{2}}{\cos \frac{A+B}{2}} \tan \frac{c}{2} \qquad (3\text{-}9)$$

$$\tan \frac{a-b}{2} = \frac{\sin \frac{A-B}{2}}{\sin \frac{A+B}{2}} \tan \frac{c}{2} \qquad (3\text{-}10)$$

$$\tan \frac{A+B}{2} = \frac{\cos \frac{a-b}{2}}{\cos \frac{a+b}{2}} \cot \frac{C}{2} \qquad (3\text{-}11)$$

$$\tan \frac{A-B}{2} = \frac{\sin \frac{a-b}{2}}{\sin \frac{a+b}{2}} \cot \frac{C}{2} \qquad (3\text{-}12)$$

Eq. 3-6 is called the analog to the cosine formula; Eqs. 3-7 through 3-12 are called half-angle formulas. Eqs. 3-9 and 3-10 are usually solved as a pair to yield values for the sides a and b; similarly, Eqs. 3-11 and 3-12 are solved as a pair to give the angles A and B.

3.2 Right Spherical Triangles. If, in a spherical triangle, one of the angles is equal to 90°, the triangle is a right spherical triangle. All of the formulas given in the preceding article for oblique spherical triangles also hold for right spherical triangles; it may be more convenient, however, to use formulas which have been developed especially for the right triangle.

Let the right angle in a right spherical triangle be denoted by the letter C, and let the five remaining parts of the triangle be indicated in the manner and order as shown in Fig. 3.3. This is also the order in which they appear in the triangle if the right angle is omitted. Any one of the five parts of the circular diagram may be selected and called a middle part; then the two parts of the diagram next to it on each side are called adjacent parts and the other two opposite parts. Two fundamental rules, known as Napier's Rules of Circular Parts, may be shown as true; they are as follows:

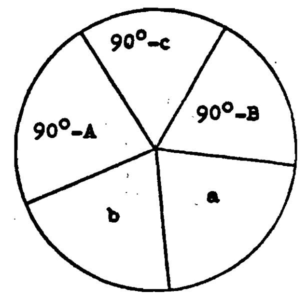


Fig. 3.3 Napier's Rules of Circular Parts.

* (1) The sine of a middle part is equal to the product of the cosines of the opposite parts.

* (2) The sine of a middle part is equal to the product of the tangents of the adjacent parts.

Using these rules, ten different formulas may be written involving relations between the various parts of the original spherical triangle.

Example. In the right spherical triangle shown in Fig. 3.4, find the missing side a.

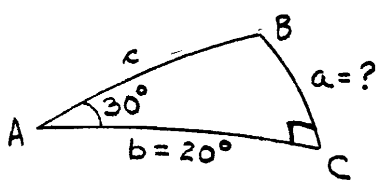


Fig. 3.4

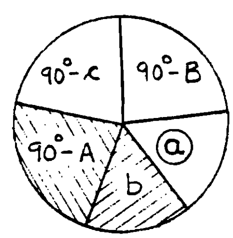


Fig. 3.5

In Fig. 3.5, the shaded sections represent the known values and the circled quantity represents the desired unknown value. Using rule 2, with the quantity b selected as a middle part, we may write

$$\sin b = \tan(90°-A)\cdot\tan a \quad \rightarrow \frac{1}{\tan(90°-A)} = \tan A$$

which after rearranging becomes

$$\tan a = \sin b \tan A$$

$$= \sin 20° \tan 30°$$

$$\therefore a = 11° \, 10' \, 12.''8$$

* TACOS = Tan products of Adjacent sides = Cos product of Opposite sides = S products of middle side

<u>3.3 Polar Triangles.</u> Given the spherical Δ ABC as shown in Fig. 3.6, locate A', B', and C', the poles of the sides a, b, and c. Draw the polar spherical Δ A' B' C'.

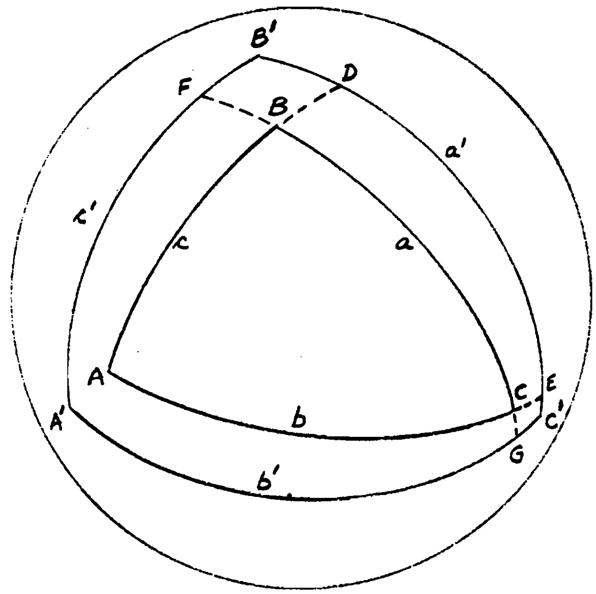


Fig. 3.6 Polar triangles

Since A is 90° from both B' and C', it follows that A is the pole of side B' C' = a'; thus A, B, and C are the poles of the sides of the polar triangle.

B' E = 90° since B' is the pole of the great circle ACE.
DC' = 90° since C' is the pole of the great circle ABD.
Adding, B' E + DC' = 180°
B' E + DE + EC' = 180°
But B' C' = the side a', and DE = the angle A, so that
a' + A = 180°

∴ Any angle of a spherical triangle is the supplement of the corresponding side of the polar triangle.

FC = 90°, since C is the pole of the great circle A' F B'.
BG = 90°, since B is the pole of the great circle A' G C'.
Adding, FC + BG = 180°
FG + BC = 180°
A' + a = 180°

∴ Any side of a spherical triangle is the supplement of the corresponding angle of the polar triangle.

∴ Any side or angle of a spherical triangle is the supplement of the corresponding angle or side of the polar triangle.

The above results may be used in conjunction with Eqs. 3-1 through 3-5 to yield new formulas. For example, the Law of Cosines (Eq. 3-1) applied to the polar triangle of Fig. 3.6 becomes

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'$$

$$\cos (180-A) = \cos (180-B) \cos (180-C) + \sin (180-B) \sin (180-C) \cos (180-a)$$

$$-\cos A = -\cos B (-\cos C) + \sin B \sin C (-\cos a)$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a \qquad (3-13)$$

3.4 General Remarks on Spherical Triangle Solution. A spherical triangle has six elements, or parts: the three sides and the three vertex angles. If any three of these parts are given, the other three can be found by using one or more formulas of spherical trigonometry. The choice of formula may be based upon several criteria, such as: computing power available, what parts are given, what parts are to be found, and the degree of accuracy desired in the solution. By proper choice of formula, a direct solution can usually be made, that is, the desired part can be solved for by means of a single formula. In some cases it may be preferable or necessary to use the indirect method of finding a missing, though undesired, part first and then solving for the desired part through use of a second formula.

The following reminders and cautions may be helpful:

(1) An angle near 0° is best found through its sine or tangent.
(2) An angle near 90° is best found through its cosine or tangent.
(3) The sine of an angle is always positive for the range $0° \leq \theta \leq 180°$, so that it becomes necessary to resolve the ambiguity of quadrant by independent means, when solving for the angle through its arcsine.
(4) The cosine and the tangent of an angle are both positive for the range $0° \leq \theta \leq 90°$ and negative for the range $90° \leq \theta \leq 180°$, so that there is no ambiguity of quadrant.
(5) The sum of the angles of a spherical triangle is always greater than 180° and less than 540°; the amount by which the sum of the angles exceeds 180° is called the spherical excess of the triangle. The spherical excess approaches zero when the sides of the spherical triangle are very small compared with the radius of the sphere; it approaches 360° when the opposite is true.
(6) The sum of the sides of a spherical triangle is always less than 360°.

3.5 The Astronomical Triangle. The spherical triangle having the elevated pole, the zenith, and a heavenly body (or an apparent point on the celestial sphere) as the three vertices is called the astronomical triangle and is of great importance in practical astronomy, since many of the problems encountered will require solution by means of it. There are four possible cases to consider:

(1) observer in north latitude, body west of the meridian
(2) " " " " , " east " " "
(3) " " south " , " " " " " , and
(4) " " " " , " west " " " .

The first of these cases is shown in Fig. 3.7, and may be considered as typical, since the basic rules for the construction and solution of all four cases are the same. In the figure, P represents the elevated celestial pole (the celestial north pole in this case); the observer's zenith is at Z, and the heavenly body (or the point of interest on the celestial sphere) is at B.

By reference to the figure at right, it is seen that the arc PZ is the complement of the latitude, and the arc PB is the complement of the declination; these two arcs are often called the co-latitude and the co-declination, respectively. The arc PB is also known as the polar distance. The arc ZB is the zenith distance z, and is the complement of the altitude a.

The angle at the pole between the meridian and the hour circle through the body is called the meridian angle μ. The meridian angle is numerically equal to the local hour angle when the body is west of the meridian; however, when the body is east of the meridian, the meridian angle is equal to 360° minus the hour angle. The angle Z at the zenith, between the meridian and the vertical circle through the body, is equal to the azimuth A for the case when the body is east of the meridian, but becomes 360°- A when the body is west of the meridian.

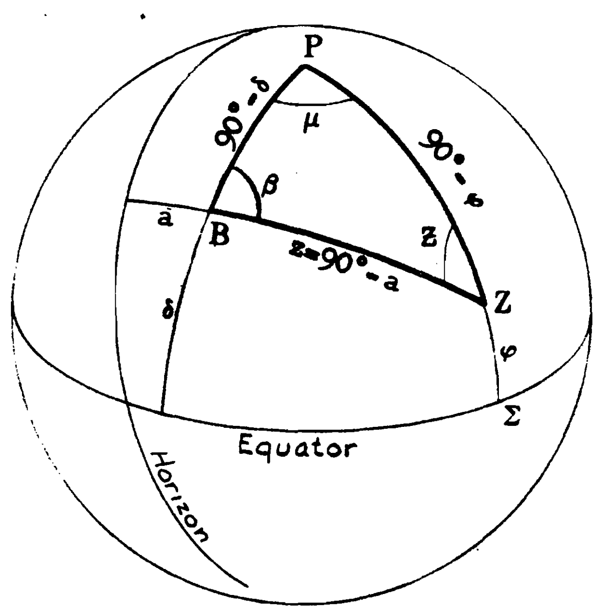


Fig. 3.7 The astronomical triangle for the case of an observer at north latitude with the body west of the meridian.

The parts of the astronomical triangle are:

Sides:	polar distance		$90° - \delta$
	zenith distance	$z =$	$90° - a$
	co-latitude		$90° - \varphi$
Angles:	meridian angle	μ	at the pole
	zenith angle	Z	at the zenith
	parallactic angle	β	at the body (this angle is normally not needed in the solution of the astronomical triangle)

Observe that the angle at the pole is measured by the corresponding arc of the equator and the angle at the zenith by the corresponding arc of the horizon.

3.6 Solution of the Astronomical Triangle. If any three of the six elements or parts of the astronomical triangle are known, the remaining parts may be found by means of the formulas given in sections 3.1 through 3.3 .

Case 1: Observer in north latitude, body west of the meridian (Fig. 3.8).

The usual situation is that the latitude, declination, and hour angle are known, and that the azimuth and altitude are to be found.

Writing the four-consecutive-parts formula (Eq. 3-5) for the zenith angle Z, which in this case is equal to 360°- A,

$$\tan(360°-A) = \frac{\sin h}{\frac{\sin(90°-\varphi)}{\tan(90°-\delta)} - \cos(90°-\varphi)\cos h} \quad (3\text{-}14)$$

From plane trigonometry, the following relations are true:

$$\tan(360°-\theta) = -\tan\theta$$
$$\sin(90°-\theta) = \cos\theta$$
$$\cos(90°-\theta) = \sin\theta$$
$$\tan(90°-\theta) = \cot\theta = \frac{1}{\tan\theta}$$

Using these relations in Eq. 3-14,

$$\tan A = \frac{-\sin h}{\cos\varphi\tan\delta - \sin\varphi\cos h} \quad (3\text{-}15)$$

Writing the law of cosines (Eq. 3-1) for the side z°= 90°- a,

$$\cos(90°-a) = \cos(90°-\varphi)\cos(90°-\delta) + \sin(90°-\varphi)\sin(90°-\delta)\cos h$$

$$\sin a = \sin\varphi\sin\delta + \cos\varphi\cos\delta\cos h \quad (3\text{-}16)$$

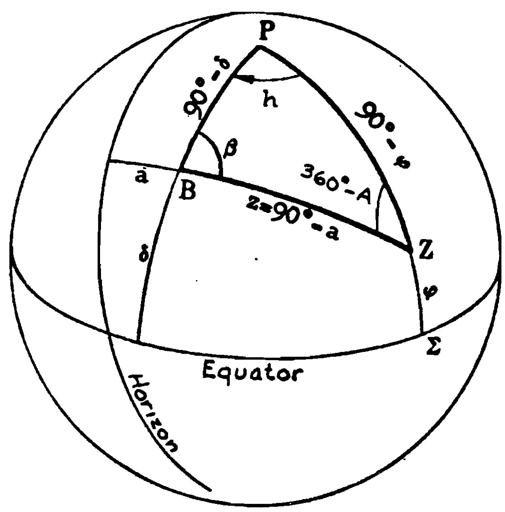


Fig. 3.8 The astronomical triangle for the case of an observer at north latitude with the body west of the meridian.

Case 2: Observer in north latitude, body east of the meridian (Fig. 3.9).

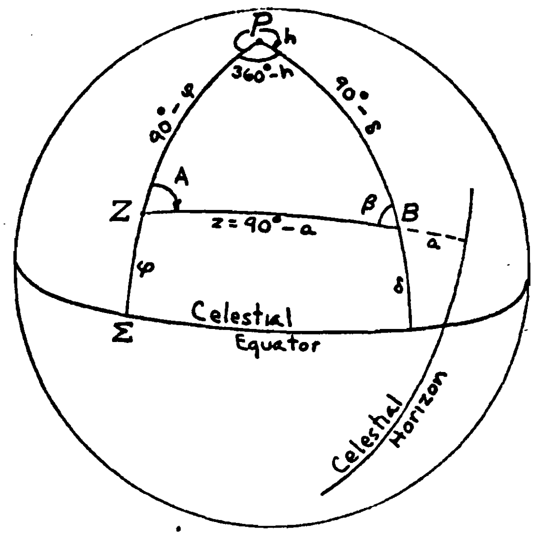


Fig. 3.9 The astronomical triangle for the case of an observer in north latitude with the body east of the meridian.

The parts of the astronomical triangle are:

Sides:	polar distance	$90° - \delta$
	zenith distance	$z = 90° - a$
	co-latitude	$90° - \varphi$
Angles:	meridian angle	$360° - h$
	zenith angle	A
	parallactic angle	β

Again, suppose that the latitude, declination, and hour angle are known, with the azimuth and altitude required.

Writing the four-consecutive-parts formula for the zenith angle, which here is equal to the azimuth A,

$$\tan A = \frac{\sin(360° - h)}{\frac{\sin(90° - \varphi)}{\tan(90° - \delta)} - \cos(90° - \varphi)\cos(360° - h)}$$

From plane trigonometry, $\sin(360° - \theta) = -\sin\theta$, and $\cos(360° - \theta) = \cos\theta$, so that the above expression may be written as

$$\tan A = \frac{-\sin h}{\cos\varphi\tan\delta - \sin\varphi\cos h} \qquad (3\text{-}15)$$

which is the same as for Case 1.

Writing the law of cosines for the side $z = 90° - a$,

$$\cos(90° - a) = \cos(90° - \varphi)\cos(90° - \delta) + \sin(90° - \varphi)\sin(90° - \delta)\cos(360° - h)$$

$$\sin a = \sin\varphi\sin\delta + \cos\varphi\cos\delta\cos h \qquad (3\text{-}16)$$

which is the same as for Case 1.

It may be shown that the formulas for Cases 3 and 4 are the same as for Case 1, so that Eqs. 3-15 and 3-16 are valid in all cases.

3.6.1 When working toward the solution of other astronomical triangles which involve different combinations of known and desired quantities than those discussed in the preceding pages, additional formulas may be needed; several are given below, without proof. For the sake of compactness, the meridian angle (at the pole) is in all cases denoted by the symbol μ, and the zenith angle by Z; remember that the meridian angle is equal to either the hour angle or 360° minus the hour angle, and that the zenith angle is equal to either the azimuth or 360° minus the azimuth, depending upon the location of the body relative to the meridian.

From the law of sines, Eq. 3-3, we have

$$\frac{\sin \mu}{\sin(90^\circ - a)} = \frac{\sin Z}{\sin(90^\circ - \delta)}$$

which, using the co-function relations, may be written as

$$\sin \mu \cos \delta = \cos a \sin Z \qquad (3\text{-}17)$$

In the solution of this formula for either μ or Z, it should be noted that the resulting angle lies either in the first or second quadrant, i.e., the numerical value lies in the range $0^\circ \leq \theta \leq 180^\circ$.

Again making use of the law of cosines, but writing it this time for the side $90^\circ - \delta$, we obtain the formula

$$\sin \delta = \sin \varphi \sin a + \cos \varphi \cos a \cos A \qquad (3\text{-}18)$$

For computing μ, the following additional formulas may be helpful:

$$\sin \frac{\mu}{2} = \sqrt{\frac{\cos(s+\varphi)\cos(s+\delta)}{\cos \varphi \cos \delta}} \qquad (3\text{-}19)$$

$$\text{where} \quad s = \frac{270^\circ - (\varphi + \delta + a)}{2}$$

or:

$$\tan \mu = \frac{\sin Z}{\cos \varphi \tan a - \sin \varphi \cos Z} \qquad (3\text{-}20)$$

For computing Z, the following formula may be used:

$$\sin \frac{Z}{2} = \sqrt{\frac{\cos(s+\varphi)\cos(s+a)}{\cos \varphi \cos a}} \qquad (3\text{-}21)$$

$$\text{where} \quad s = \frac{270^\circ - (\varphi + \delta + a)}{2}$$

All the formulas presented are valid for both hemispheres; they are also valid for the situation in which the zenith and the body are in different hemispheres. They are all based upon the north celestial pole being the pole point in the astronomical triangle.

3.7 Transformation from ecliptic to equatorial coordinates.

It often happens that the ecliptic coordinates λ, β of a heavenly body are known and that the corresponding independent equatorial coordinates α, δ are required. The transformation equations may be obtained by considering Figure 3.10, which represents the celestial sphere, with P the north celestial pole and P_E the north ecliptic pole. The equator and ecliptic are shown, as are the vernal equinox ♈, the summer solstice SS, the foot F of the hour angle of the solstice, and the obliquity of the ecliptic, denoted as ϵ. The arc F-SS is equal to the obliquity ϵ, as is the arc P-P_E.

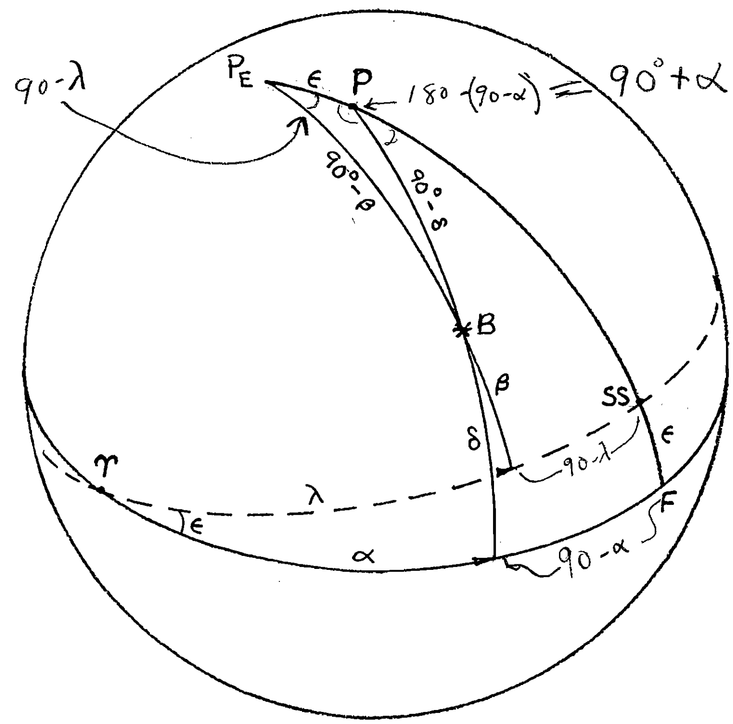


Fig. 3.10

In the spherical triangle PBP_E, the angle at P_E is equal to $90^\circ - \lambda$, and the angle at P is equal to $180^\circ - (90^\circ - \alpha)$. From the four consecutive parts formula,

$$\tan(90^\circ + \alpha) = \frac{\sin(90^\circ - \lambda)}{\dfrac{\sin\epsilon}{\tan(90^\circ - \beta)} - \cos\epsilon\cos(90^\circ - \lambda)}$$

$$-\frac{1}{\tan\alpha} = \frac{\cos\lambda}{\sin\epsilon\tan\beta - \cos\epsilon\sin\lambda}$$

$$\tan\alpha = \frac{\cos\epsilon\sin\lambda - \sin\epsilon\tan\beta}{\cos\lambda} \qquad (3\text{-}21)$$

From the law of cosines,

$$\cos(90^\circ - \delta) = \cos\epsilon\cos(90^\circ - \beta) + \sin\epsilon\sin(90^\circ - \beta)\cos(90^\circ - \lambda)$$

$$\sin\delta = \cos\epsilon\sin\beta + \sin\epsilon\cos\beta\sin\lambda \qquad (3\text{-}23)$$

Exercises

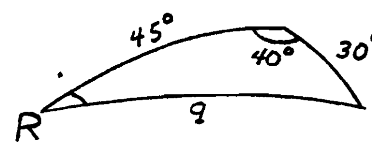
3-1. Three points on a sphere are located equidistant from each other, forming an equilateral spherical triangle; as in plane trigonometry, such a triangle is also equiangular. Derive a formula for the vertex angle as a function of the side.

3-2. Using the formula derived for problem 3-1, find the vertex angle corresponding to a side of (a) 90° (b) 120°. Sketch these two triangles on a globe.

3-3. Again referring to problem 3-1, what is the minimum possible vertex angle, and the corresponding side? the maximum?

3-4. May a spherical triangle have one and only one of its elements to be 90°? two only? three? four? five? May all six elements be 90°?

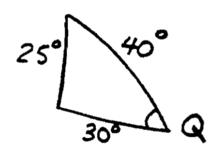
3-5. Same as 4, but for 180° elements.



3-6. A spherical triangle has certain parts given as shown in the figure at right; find side q.

3-7. For the same figure, find the vertex angle R.

3-8. Using the value of q as found in problem 3-6, verify the value for R found in problem 3-7.



3-9. Repeat problem 3-8, using a different formula.

3-10. In the figure at right, find the vertex angle Q.

3-11. A spherical triangle has one vertex angle of 20°, another of 90°, and the side included between them is 40°. Find all other parts of the triangle.

3-12. A spherical triangle has one side of 20°, another of 90°, and the vertex angle included between them is 40°. Find all other parts of the triangle.

3-13. Given $\varphi = 55°N$, LHA = 3^h, and $\delta = 36°N$, find the azimuth and altitude.

3-14. Find the hour angle and declination of a star if its azimuth is 230° and its altitude is 40° at a place in latitude 45°N.

3-15. Find the azimuth and altitude of Procyon for a place in 30° south latitude at an instant when the LHA of the vernal equinox is $4^h\ 30^m$.

3-16. Solve exercise 23 of chapter 2 by the methods of the present chapter.

3-17. Same, but for exercise 2-25.

3-18. Same, but for exercise 2-26.

3-19. Same, but for exercise 2-27.

3-20. Find the azimuth of Betelgeuse at the time of its setting as seen from (a) 30° S latitude (b) 30° N latitude.

3-21. Convert the ecliptic coordinates $\lambda = 342°13'17''$, $\beta = +7°51'22''$ to independent equatorial coordinates, for Nov. 3 at 0^h TDT.

3-22. A certain star is located at $\alpha = 5^h\ 20^m$, $\delta = -7°\ 14'$; find its ecliptic coordinates at the epoch Feb. 27 at 0^h TDT.

3-23. A planet is at $\lambda = 276°\ 19'\ 46''$, $\beta = +2°\ 4'\ 6''$; if the hour angle of the vernal equinox is $21^h\ 17^m\ 12^s$, find the azimuth and altitude of the planet as seen from latitude 15° 40' 20" S at the epoch July $19^d\ 15^h\ 20^m$ TDT.

$18^h\ 3^m$ TDT

3-24. Venus is to be observed from EMOS on April 17 at the epoch specified by the instructor. Find its expected azimuth and altitude. The hour angle of the vernal equinox, needed for solution of this problem, will be furnished by the instructor also. $A = 191.°~~7046~~\ 8190119$
$LHA_\gamma = 0°\ 40^m\ 5.^s5051$

3-25. Convert the ecliptic longitude and latitude of the moon as given in the Astronomical Almanac for the epoch October $8^d\ 0^h$ Dynamical Time to right ascension and declination. Compare your results with the tabular values of r.a. and dec. also given in the Almanac.

3-26. Repeat the preceding problem, but for the sun.

CHAPTER 4

TIME AND LONGITUDE

4.1 Time Systems. As was mentioned in the preceding chapter, the celestial coordinates of heavenly bodies such as the sun, moon, stars, and planets change with respect to time. In order to discuss these changes in some detail, and to understand their effect on the calculations and observations of engineering astronomy, it is necessary to become familiar with several of the various types of time systems in use today.

It is necessary first to distinguish between two different aspects of time: the epoch, which is the instant of occurrence of some phenomenon or observation, and the interval, which is the time elapsed between two epochs. The conventional time scale is measured in units of years, months, days, hours, minutes, and seconds.

A fundamental requirement of any time measurement system is to establish a relationship between the units of measurement and some observable physical event which is either repetitive and countable, or continuous and measurable, or both. The systems which we will use are based either upon observable astronomical phenomena such as star transits, the diurnal motion of the sun, etc., or upon observable physical phenomena involving certain properties of the cesium atom.

There are three systems of the first type with which we must familiarize ourselves, as listed below and as discussed in some detail in the articles which follow. The first two are based upon the diurnal rotation of the earth, with the third being based upon the annual orbital motion of the earth about the sun. These three systems are:

(1) Sidereal time, or time measured with respect to the apparent motion of the stars; this apparent motion is actually due to the diurnal rotation of the earth upon its axis.

(2) Solar time, or time measured with respect to the apparent motion of the sun; this apparent motion is also actually due to the diurnal rotation of the earth.

(3) Dynamical time, formerly called ephemeris time, which is the independent variable in the equations of motion of celestial mechanics; its measure is defined by the orbital motion of the earth around the sun.

The fourth, non-astronomical, time system which we will use is:

(4) Atomic time, based upon analyses of certain physical properties of the cesium 133 atom.

4.2 Dynamical Considerations. Before these time systems are described more fully, there are two phenomena which need to be understood in principle, at least. These are (a) the motion of the equinox among the stars and (b) variations in the rotational speed of the earth.

The vernal equinox, heretofore considered as fixed with respect to the stars, is actually in continuous, though slow, motion because of dynamical forces on the earth as it both rotates on its axis and revolves about the sun, which cause the equator and the ecliptic, and hence the equinox, to move relative to the celestial sphere.

The motion of the earth's equator, and therefore of the celestial poles, is due to the gravitational attraction of the sun and the moon upon the earth's equatorial bulge, and may be resolved into two components. The first component, the luni-solar precession, moves the celestial poles about the ecliptic poles in a circular path with a period of about 25,800 years and an amplitude equal to the obliquity of the ecliptic, resulting in a westerly movement of the equinox along the equator of about 50" per year (a more precise value is given in the Almanac, on page K6). The second component, called the nutation, and caused in part by the fact that the plane of the moon's orbit about the earth is not in the ecliptic plane, consists basically of a periodic motion of the celestial poles (superimposed on the luni-solar precession) having a maximum amplitude of about 9" and a main period of about 18.6 years.

The motion of the ecliptic is due to the gravitational attraction of the planets upon the earth as a whole, and consists of a slow rotation of the ecliptic about a slowly-moving diameter, resulting in a westerly movement of the equinox of about 47" per century. This effect is called the planetary precession.

The rotational speed of the earth itself is not perfectly uniform, but is subject to irregularities of three types:

(a) seasonal or periodic variation, more or less repeatable from year to year, probably due to atmospheric and tidal effects;

(b) a secular (non-periodic) decrease in rotational speed, due chiefly to energy-dissipative tidal forces; and

(c) irregular fluctuations, the causes of which are only partly understood.

It should also be pointed out that there is a continuous, although slight, motion of the solid mass of the earth itself relative to the axis of rotation. This effect, known as the wandering of the poles, or simply as the polar motion, causes the geographical coordinates of the observer to change with time.

4.3 Sidereal Time. Sidereal time is directly related to the diurnal rotation of the earth; equal intervals of angular motion correspond to equal intervals of sidereal time. At the instant of upper transit of the vernal equinox over a particular meridian, the sidereal time on that meridian is taken as $0^h\ 0^m\ 0^s$. This epoch is called sidereal apparent noon and is considered as the beginning of the sidereal day, which is defined as the interval between two successive upper transits of the vernal equinox over the same meridian. The length of the sidereal day is not constant, nor does it precisely measure the true time for one rotation of the earth, because of the movement of the equinox as described in the preceding article.

The sidereal epoch is numerically equal to the hour angle of the vernal equinox. The local hour angle of the true (apparent) equinox is called the local apparent sidereal time (LAST). The sidereal time on the Greenwich meridian is called the Greenwich apparent sidereal time (GAST). The local hour angle of the mean equinox is called the local mean sidereal time (LMST); when it is referred to the Greenwich meridian, it is called the Greenwich mean sidereal time (GMST). These four relations may be written as

LAST = LHA of the apparent equinox (4-1)
GAST = GHA " " " (4-2)

LMST = LHA of the mean equinox (4-3)
GMST = GHA " " " (4-4)

= nutation of longitude

The term equation of the equinoxes (EOE) is applied to the quantity "apparent sidereal time minus mean sidereal time," thus

EOE = LAST - LMST (4-5)
or EOE = GAST - GMST (4-6)

The equation of the equinoxes is due to the nutation (Section 4.2). The numerical value of EOE is tabulated for each day of the year in the Almanac, in the table of Universal and Sidereal Times, beginning on page B8.

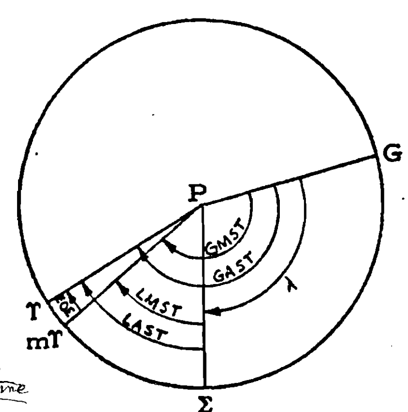
All of the above relations are illustrated in Fig. 4.1, from which it is evident that

$$\text{LMST} = \text{GMST} - \lambda \quad (4\text{-}7)$$
$$\text{LAST} = \text{GAST} - \lambda \quad (4\text{-}8)$$

or, after slight rearrangement,

$$\text{GMST} - \text{LMST} = \lambda \quad (4\text{-}9)$$
$$\text{GAST} - \text{LAST} = \lambda \quad (4\text{-}10)$$

Fig. 4.1 **Relation between the sidereal times at two places and their difference in longitude. The equation of the equinoxes is also shown, though very much enlarged for clarity.**



1,0027379093 = sidereal time / solar time

Equations 4-9 and 4-10 are specific cases of a general principle of time measurement which may be stated thus: The difference between the corresponding local times of two places is equal to their difference in longitude. This principle will be shown to apply to the other kinds of time as discussed in the following articles.

As was shown in section 2.12, the hour angle of the vernal equinox is equal to the right ascension of a body, such as a star, at the instant of its upper transit. It follows, therefore, that the sidereal time may be determined by observing transits of stars, since, from Eqs. 2-12 and 4-1, the local apparent sidereal time at any instant is equal to the right ascension of a star which is at upper transit at the same instant.

4.4 Solar Time. An apparent solar day is defined as the interval between two successive lower transits of the sun's center over the same meridian. The lower, rather than the upper, transit is used so that the change of date will occur at midnight. The apparent solar day is divided into 24 hours, beginning with the instant the center of the sun is at lower transit (local apparent midnight). The instant the center of the sun is at upper transit is known as local apparent noon; the local apparent time at that instant is 12^h. At any instant, the local apparent solar time, or simply the local apparent time, is equal to the hour angle of the sun's center plus 12 hours, that is,

$$LAT = LHA_{\odot} + 12^h \qquad (4\text{-}11)$$

The local apparent time on the Greenwich meridian is called the Greenwich apparent time (GAT), and is equal to the Greenwich hour angle of the sun's center plus 12 hours, that is,

$$GAT = GHA_{\odot} + 12^h \qquad (4\text{-}12)$$

The length of the apparent solar day is not constant, nor does it measure the true time for one complete rotation of the earth, not only because of the irregularities in the earth's rotational speed, as described in section 4.2, but also because of the way in which the earth moves about the sun. It was stated in section 1.6 that the sun appears to move eastward through the stars at a rate of about 1^o per day because of the actual annual revolution of the earth about the sun. This motion is now understood to be non-uniform, since the earth moves at varying speeds in obedience to the laws of celestial mechanics as formulated by Kepler. The non-uniform actual motion of the earth results in a non-uniform apparent motion of the sun, thus causing the apparent solar day to vary in length. Even if the sun did move uniformly, the length of the solar day would still not be constant, since the sun moves along the ecliptic whereas the rotation is measured along the equator, and angles at the celestial pole measuring equal arcs on the ecliptic are not, in general, equal.

The use of the true (apparent) sun is, for the reasons just explained, not advisable as a means for precise timekeeping. Instead, a fictitious fiducial point called the mean sun is imagined as moving eastward at a uniform rate along the equator and to complete one revolution from the vernal equinox in the same time in which the true sun completes one revolution on the ecliptic. This interval (about 365 1/4 apparent solar days) is called the tropical year; it is more rigorously defined as the interval between two successive passages of the center of the sun through the vernal equinox. A precise value of the length of the current tropical year is given on page C1 of the Almanac. The time given by the mean sun is such that every day measured by it is of exactly the same duration, and each one is equal to the average (mean) solar day as measured by the true sun. A mean solar day may then be defined as the interval between two successive lower transits of the mean sun over the same meridian. The local mean solar time, or simply the local mean time (LMT), is the hour angle of the mean sun plus 12 hours, that is,

$$LMT = LHA_{m\odot} + 12^h \qquad (4\text{-}13)$$

The local mean time on the Greenwich meridian is called either the Greenwich Mean Time (GMT) or Universal Time (UT), and is equal to the Greenwich hour angle of the mean sun plus 12 hours, that is,

$$UT = GMT = GHA_{m\odot} + 12^h \qquad (4\text{-}14)$$

Local mean noon at any place is the instant of upper transit of the mean sun over the meridian of the place; mean midnight refers to the instant of lower transit. The mean solar day is divided into 24 hours beginning at mean midnight.

Since the mean sun is not observable, it is necessary to introduce a method for changing from one kind of solar time to the other. This is done by the equation of time (EOT), which is defined as the quantity "apparent solar time minus mean solar time" (compare this expression with that for the equation of the equinoxes in section 4.3). We may then write

$$EOT = LAT - LMT = GAT - UT \qquad (4\text{-}15)$$

The equation of time is not tabulated in the Almanac, though it may be computed from data which is tabulated, using the method as explained later in this chapter. Values of the EOT are given twice daily in the Nautical Almanac, to a precision of 1^m of time. Fig. 4.2 shows graphically the equation of time for the year 1945; the graph is essentially the same for all years.

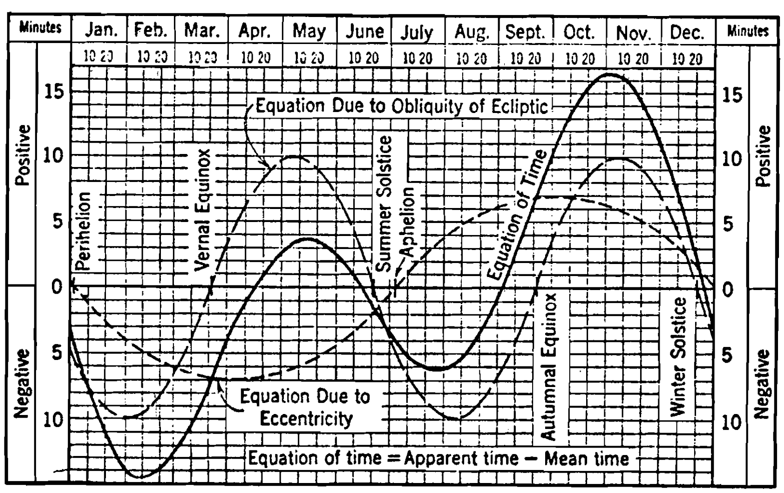


Fig. 4.2. **The equation of time.**

The relations described in the preceding paragraphs, and as given by Eqs. 4-1 through 4-15, are illustrated by Fig. 4.3, from which it is evident that

$$LMT = GMT - \lambda = UT - \lambda \qquad (4\text{-}16)$$
$$LAT = GAT - \lambda \qquad (4\text{-}17)$$

or, after slight rearrangement,

$$UT - LMT = GMT - LMT = \lambda \qquad (4\text{-}18)$$
$$GAT - LAT = \lambda \qquad (4\text{-}19)$$

again illustrating the basic principle first stated in section 4.3: The difference between the corresponding local times of two places is equal to their difference in longitude.

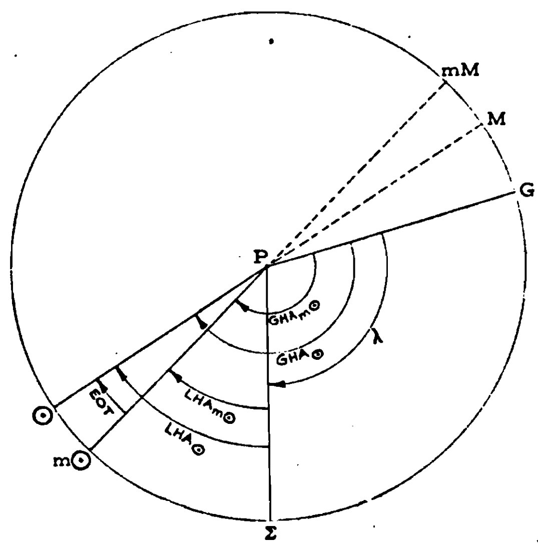


Fig. 4.3 Relation between the solar times of two places and their difference in longitude. The point labeled M is the midnight point, located 12^h ahead of the sun; similarly, mM denotes the mean midnight point, which is located 12^h ahead of the mean sun.

The definition of Universal Time as given above is for the value as deduced directly from observations; for greater precision, it is necessary to correct the observed values for the rotational irregularities of both polar motion and seasonal variation. The numbers 0, 1, and 2 are conventionally used for this purpose, in the following way:

UT0 = UT as deduced directly from observations
UT1 = UT0 corrected for polar motion
UT2 = UT0 corrected for both polar motion and seasonal variation.

The corrections for polar motion and seasonal variation are so small that it will normally not be necessary in our work to distinguish between the three types, so that we shall call any one of them simply "Universal Time," unless the topic is such that it is necessary to be specific.

4.5 Relation Between Mean Solar and Mean Sidereal Time Interval. A definite relation exists between an interval of mean solar time and the corresponding interval of mean sidereal time, so that knowing the one is equivalent to knowing the other. To understand this relationship, consider the earth moving about the mean sun as shown in Fig. 4.4.

Suppose that at a given instant the mean sun is on the meridian of an observer at O, and that a line joining the earth's center C and the mean sun S points toward a certain star. After 24 sidereal hours, the earth's center is at C' and the earth has completed a full rotation so that the star is again on the observer's meridian.

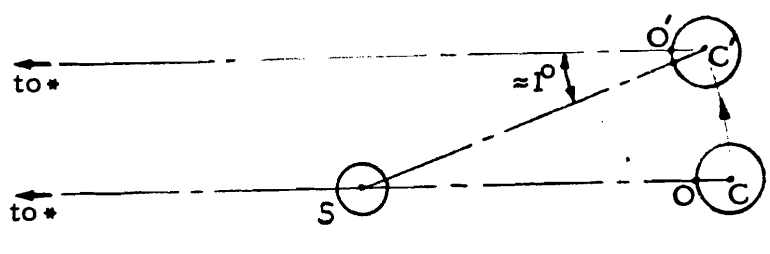


Fig. 4.4 **Relation between solar and sidereal time interval.**

From the figure, it is clear that the mean sun is not yet on the meridian, lacking about 1^{o} of rotation, which is equivalent to about 4^{m} of time. This shows that the mean sidereal day is shorter than the mean solar day by about 4^{m}. The sidereal clock thus gains on the mean solar clock by this amount daily, which accrues to a full day at the end of the tropical year.

The length of the tropical year is given in the Almanac as containing 365.24220 mean solar days (after rounding to an accuracy sufficient for our purposes); the mean sidereal year thus contains 366.24220 mean solar days, leading to the following ratio:

$$1 \text{ mean solar day} = \frac{366.24220}{365.24220} \text{ mean sideareal day} = 1.0027379093 \text{ mean sidereal day}$$

This ratio is, in general, the ratio of any interval of mean sidereal time to the corresponding interval of mean solar time. Conversion from an interval of mean sidereal time to the corresponding interval of mean solar time, or the converse, may be done by means of the above relation.

4.6 Relation Between Mean Solar and Mean Sidereal Time Epoch. Until 1984 January 1, for consistency with the FK4 system of astronomical constants in use until that time, the fundamental relation between the epoch of mean solar and mean sidereal time is given by the expression

$$\text{Greenwich mean sid. time @ } 0^h \text{ UT1} = 6^h\ 38^m\ 45\overset{s}{.}836 + 8640184\overset{s}{.}542\ T + 0\overset{s}{.}0929\ T^2 \qquad (4\text{-}20)$$

where T denotes the number of Julian centuries of 36525 days which, at the beginning of the calendar day concerned, have elapsed since 1900 January $0\overset{d}{.}5$ (JD 2415020.0) on the Greenwich meridian.

On and after 1984 January 1, for consistency with the FK5 system of astronomical constants to be introduced at that time, the fundamental relation between the epoch of mean solar and mean sidereal time is given by

$$\text{GMST at } 0^h \text{ UT1} = 6^h 41^m 50\overset{s}{.}54841 + 8640184\overset{s}{.}812866\ T_u + 0\overset{s}{.}093104\ T_u^2 - 0\overset{s}{.}0000062\ T_u^3 \qquad (4\text{-}21)$$

where T_u denotes the number of Julian centuries of 36525 days which, at the beginning of the calendar day concerned, have elapsed since 2000 January 1^d 12^h UT1 (JD 2451545.0) on the Greenwich meridian.

The table of Universal and Sidereal times in section B of the Almanac has been constructed by using the appropriate one of the above expressions. A discussion of the table is given in the 'Explanation' section of the Almanac.

Example: Find the GMST at 0^h UT1 on 1984 January 1 (JD 2445700.5) by each of the above expressions.

$$T = \frac{2445700.5 - 2415020.0}{36525} = 0.839986$$

which, when used in eq. 4-20, gives

$$\text{GMST at } 0^h \text{ UT1} = 6^h 39^m 22\overset{s}{.}6389$$

$$T_u = \frac{2445700.5 - 2451545.0}{36525} = -0.160014$$

which, when used in eq. 4-21 gives

$$\text{GMST at } 0^h \text{ UT1} = 6^h 39^m 22\overset{s}{.}7025$$

The difference of $0\overset{s}{.}0636$ in the two results is equal to the difference in right ascensions of the FK4 and FK5 systems.

4.6 <u>Relation Between Mean Solar and Mean Sidereal Time Epoch</u>. Until 1984 January 1, for consistency with the FK4 system of astronomical constants in use until that time, the fundamental relation between the epoch of mean solar and mean sidereal time is given by the expression

$$\text{Greenwich mean sid. time @ } 0^h \text{ UT1} = 6^h\ 38^m\ 45^s.836 + 8640184^s.542\ T + 0^s.0929\ T^2 \qquad (4\text{-}20)$$

where T denotes the number of Julian centuries of 36525 days which, at the beginning of the calendar day concerned, have elapsed since 1900 January $0^d.5$ (JD 2415020.0) on the Greenwich meridian.

On and after 1984 January 1, for consistency with the FK5 system of astronomical constants to be introduced at that time, the fundamental relation between the epoch of mean solar and mean sidereal time is given by

$$\text{GMST at } 0^h \text{ UT1} = 6^h41^m50^s.54841 + 8640184^s.812866\ T_u + 0^s.093104\ T_u^2 - 0^s.0000062\ T_u^3 \quad . \qquad (4\text{-}21)$$

where T_u denotes the number of Julian centuries of 36525 days which, at the beginning of the calendar day concerned, have elapsed since 2000 January $1^d\ 12^h$ UT1 (JD 2451545.0) on the Greenwich meridian.

The table of Universal and Sidereal times in section B of the Almanac has been constructed by using the appropriate one of the above expressions. A discussion of the table is given in the 'Explanation' section of the Almanac.

Example: Find the GMST at 0^h UT1 on 1984 January 1 (JD 2445700.5) by each of the above expressions.

$$T = \frac{2445700.5 - 2415020.0}{36525} = 0.839986$$

which, when used in eq. 4-20, gives

GMST at 0^h UT1 = $6^h39^m22^s.63$~~89~~9144

$$T_u = \frac{2445700.5 - 2451545.0}{36525} = -0.160014$$

which, when used in eq. 4-21 gives

GMST at 0^h UT1 = $6^h39^m22^s.70$~~35~~31

The difference of $0^s.0636$ in the two results is equal to the difference in right ascensions of the FK4 and FK5 systems.

4.7 Ephemeris Time.* The failure of the rotational time systems (sidereal and solar) to provide a means of obtaining a uniform time scale directly from observations led astronomers to a new system in which the dynamical motion of the bodies in the solar system are used in determining the time scale. Ephemeris Time is the uniform measure of time defined by the laws of dynamics and is determined in principle from the orbital motions of the planets, specifically that of the earth. The value of Ephemeris Time at any epoch is obtained by directly comparing observed positions of the sun, moon, and planets with their gravitational ephemerides; observations of the moon are the most effective for this purpose, due to its relative nearness and greater apparent speed. An accurate determination requires observations over an extended period of time and, in practice, takes the form of determining the correction ΔT that must be added to UT to obtain ET, thus:

$$ET = UT + \Delta T \qquad (4\text{-}22)$$

The value of ΔT is not predictable, except roughly, because of the unpredictable variations in the rate of rotation of the earth. Precise values of ΔT are generally unavailable for several years after the observations are made due to the time required for data reduction and publication. The Almanac gives values of ΔT for the past several years, together with an extrapolated value or values for the current year.

Ephemeris Time is the argument in several Almanac tables of positions of the various heavenly bodies.

4.8 Atomic Time. Within the past few decades, various time scales based upon certain phenomena involving the cesium 133 atom have been devised. The particular time scale resulting from analyses by the BIH (Bureau International de l'Heure, located in Paris) of the atomic time standards of many countries is known as TAI (Temps Atomique International). The fundamental unit of TAI is identical to the fundamental unit of time in the International System of Units, that is, the SI second.

4.8.1 Coordinated Universal Time (UTC). The present trend of universal times UT0, UT1, and UT2 is that of a gradual departure from TAI, due to the gradual slowing of the earth's rotation, as shown in Fig. 4.5. To provide a world-wide, uniform time scale which has the same rate as TAI but yet which is close to UT in epoch, the time scale called Coordinated Universal Time (abbreviated as UTC) has been devised. UTC is a step-function time scale, as shown in Fig. 4.5, having the same rate as TAI (and ET, since the rates of TAI and ET are identical, as far as is known at present, to extremely high precision), but with occasional 1-second steps, called leap-seconds, to maintain agreement with the epoch of UT.

* Ephemeris Time has been re-named as Dynamical Time. All remarks made here concerning Ephemeris Time are to be regarded as pertaining to Dynamical Time. There are two kinds of Dynamical Time; they are explained in the Almanac, Section B.

Insertion of leap-seconds is done, under international agreement, by the BIH, following advance notice of several weeks or even months. The number of leap-seconds extant at any given instant is denoted as

$$\Delta AT = TAI - UTC \tag{4-23}$$

A table of leap-seconds versus past epochs is given on page B5 of the Almanac; also on that page is given a table of

$$\Delta ET = ET - UTC \tag{4-24}$$

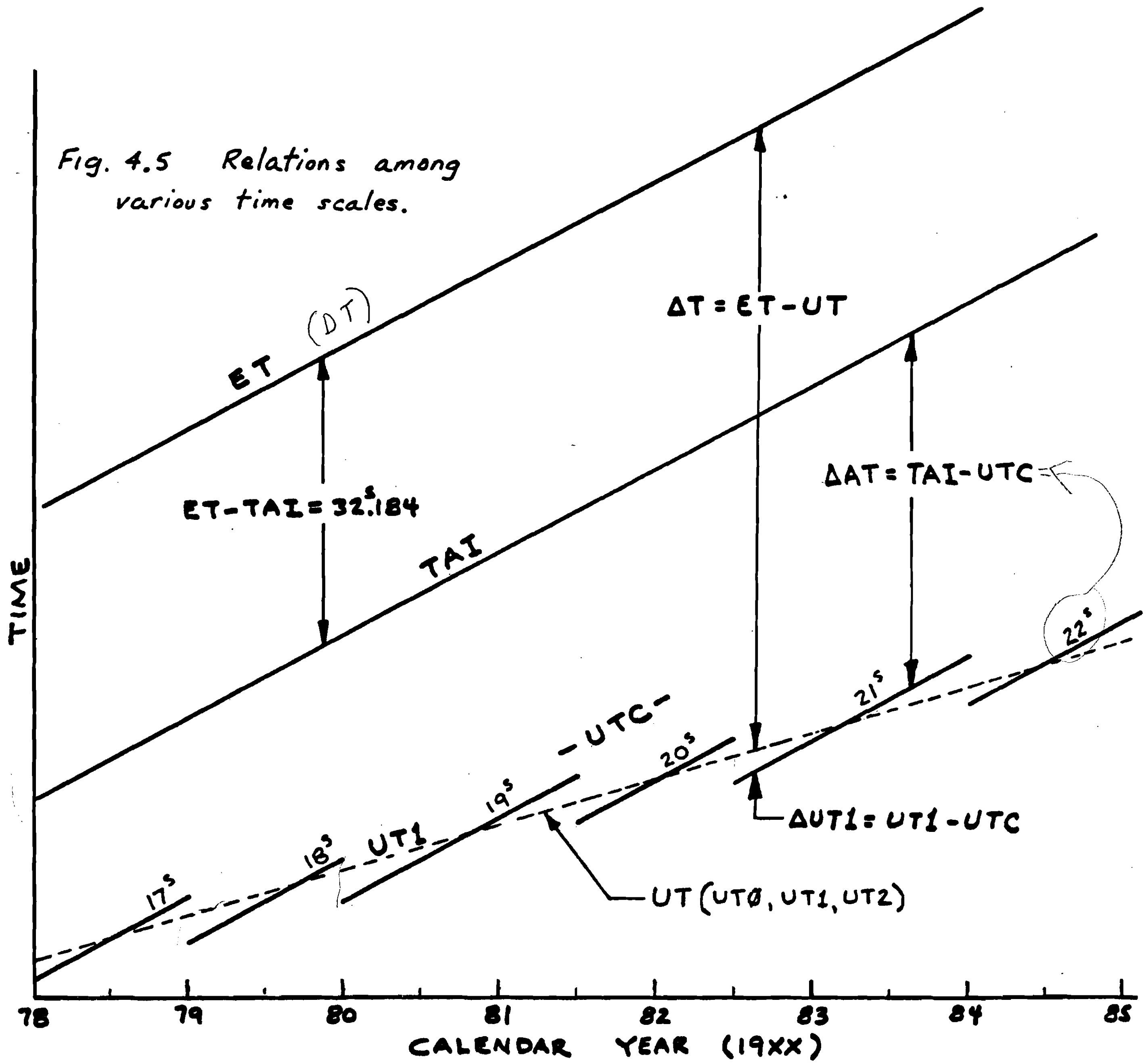


Fig. 4.5 Relations among various time scales.

4.8.2 Radio Broadcast of Time Signals. The National Bureau of Standards radio station WWV, located at Ft. Collins, Colorado, broadcasts UTC and certain other time information on a continuous basis, using frequencies of 2.5, 5, 10, 15, and 20 MHz. The transmissions include, but are not limited to, voice announcements of UTC, a binary coded decimal (BCD) time code, seconds ticks, standard audio frequencies, weather information, and UT1-UTC corrections.

4.8.3 ΔUT1 = UT1 - UTC. While UTC is the preferred time scale for ordinary, everyday, time-keeping or time-tagging purposes, including the setting of our normal clocks and watches, there are some applications such as precise navigation and satellite tracking which require that one of the universal time scales (UT0, UT1, or UT2; not UTC) which are locked to the earth's actual rotation rate be used. The particular time scale used in several of the Almanac tables (for example, the table of Universal and Sidereal times in section B) is UT1 as described in section 4.4 of the present text. To convert from UTC to UT1, a knowledge of

$$\Delta \mathrm{UT1} = \mathrm{UT1} - \mathrm{UTC} \qquad (4\text{-}25)$$

is required.

For users needing ΔUT1 to a precision of only $0\overset{s}{.}1$, WWV encodes this information into the broadcasts by using double ticks after the start of each minute. The amount of the correction is determined by counting the number of double ticks heard each minute and noting their position. The 1st through the 8th ticks indicate a positive value of UT1; the 9th through the 16th ticks indicate a negative value. For example, if the 1st, 2nd, and 3rd ticks are double, UT1-UTC = $+0\overset{s}{.}3$. Again, if the 9th and 10th ticks are double, UT1-UTC = $-0\overset{s}{.}2$.

For users requiring ΔUT1 to higher precision, or for epochs other than the current epoch, reference may be made to page B5 of the Almanac, keeping in mind that the notation "UT" as used there is strictly UT1. It will be seen that the ΔUT1 values printed there are given to a precision of $0\overset{s}{.}01$ for epochs earlier than about 1 year prior to the publication date, and to $0\overset{s}{.}1$ thereafter.

For users requiring ΔUT1 to still higher precision for current, recent past, and short-term future values, reference may be made to various publications of the U. S. Naval Observatory in Washington, D.C. For example, their "Earth Orientation Bulletin, Time Services Publication, Series 7", issued weekly to authorized recipients, contains such information to a precision of $0\overset{s}{.}0001$.

In any case, once UTC and ΔUT1 are known, UT1 is found by adding

$$\mathrm{UT1} = \mathrm{UTC} + \Delta \mathrm{UT1} = \mathrm{UTC} + (\mathrm{UT1} - \mathrm{UTC}) \qquad (4\text{-}26)$$

4.9 Time Zones. In timekeeping for ordinary civil purposes, confusion would arise if every place used the local solar time of its own meridian. In order to avoid having a different time at practically every city, a system of standard time zones has been devised, with all persons within a single zone keeping the same clock time. The world is divided into 24 zones, each having a nominal width of 15^{o} (1^{h}) of longitude (sometimes the actual border of a zone is modified to make it conform to a geographical feature or political boundary). Each zone is centered on a meridian which is an integral multiple of 15^{o}, and the zone time for each zone is taken as the local mean time for that central meridian. The same clock time is thus kept by all persons within a large area, and this time usually differs from the local mean time of any place in the zone by less than 30^{m}. The meridian of Greenwich is taken as the center of the zone system and of the zone numbered zero. Zones to the west are numbered +1, +2, +3, etc., and those to the east are numbered -1, -2, -3, etc., up to 12 in both directions. The twelfth zone is divided into two parts by the date line (the meridian 180^{o} from Greenwich), the eastern half being numbered +12 and the western half -12. When the date line is crossed in a westerly direction, the calendar date must be advanced by 1^{d}.

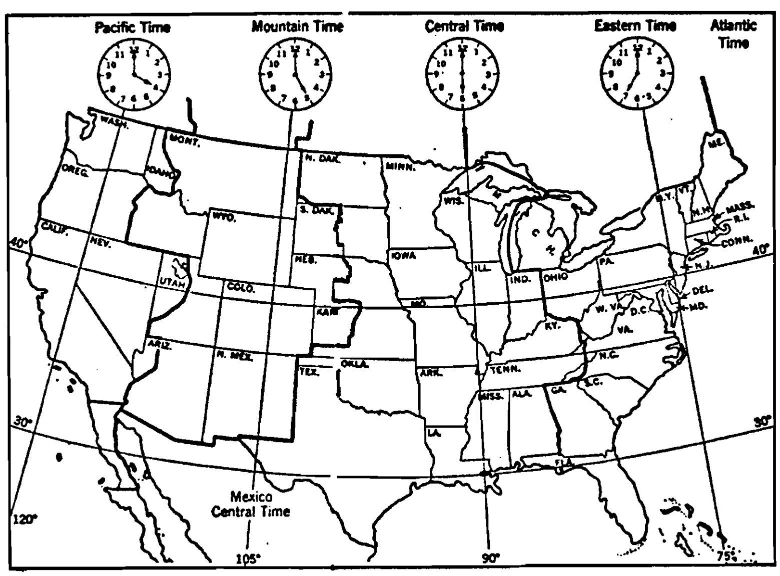


Fig. 4.4 **Map showing the standard time zones in the United States. (The clocks show the zone times corresponding to the instant of 12^{h}U.T.)**

The relation between the time on the Greenwich meridian, the time of a given zone, and the zone number or zone description is given by

$$UTC = ZT + ZD \qquad (4\text{-}27)$$

Many of the standard time zones have been given names, a few of which are listed in Table 4.1.

Table 4.1. **Standard Time Zones**

Standard Meridian	Zone Description	Zone Name	Abbreviation for the zone time
60	+4	Atlantic	AST
75	+5	Eastern	EST
90	+6	Central	CST
105	+7	Mountain	MST
120	+8	Pacific	PST
135	+9	Yukon	YST
150	+10	Alaska-Hawaii	HST
165	+11	Bering	BST

In the United States, under the Uniform Time Act of 1966, all states, The District of Columbia, and U. S. possessions must observe Daylight Saving Time beginning at $2^h\ 00^m$ Zone Time on the last Sunday in April and ending at $2^h\ 00^m$ Zone Time on the last Sunday in October, provided, however, that any state may exempt itself from the law by legislative action each year. (The states which generally do so are Arizona, Indiana, and Hawaii.) Daylight Saving Time is achieved by advancing the clock one hour; when this is done, the time then kept by each zone is the standard time of the next zone to the east, and the letter "S" in the appropriate zone time abbreviation is replaced by the letter "D". For example, in zone 7, the instant on a certain day which would normally be designated as $16^h\ 40^m$ MST will, under the Uniform Time Act, be designated as $17^h\ 40^m$ MDT; this instant is also properly designated as $17^h\ 40^m$ CST. Thus it is seen that, during the half-year in which Daylight Saving Time is in effect, persons in zone 7 keep the standard time of zone 6, those in zone 6 keep that of zone 5, and so on.

4.10 <u>Time Diagrams</u>. To aid the process of visualizing the relations between the several kinds of time, a north polar view of the celestial sphere showing the hour circles, bodies, and reference points involved may be helpful. Such a <u>time diagram</u> is shown in Fig. 4.7; PG represents the Greenwich meridian, so that PD is the date line; imagine the points G, Σ, and D as fixed, with the other points moving clockwise (i.e., westerly) around the diagram, at their proper relative speeds. The mean midnight point mM is always directly opposite the mean sun m⊙. At the instant for which the diagram is drawn, mM is between the meridians of Greenwich and the observer so that the date is different at those two places. Notice that the date changes at two places: at the mean midnight point and at the date line. There can thus be at most two calendar days in existence at the same epoch, considering the whole earth, and this is the usual case; for one instant on each day, the one day exists all around the earth. For example, the local mean time at the date line is shown as approximately $15^h\ 35^m$; after another $8^h\ 25^m$ of mean solar time, the point mM will be at the date line, and the calendar day of Monday, April 6, will exist over the entire earth.

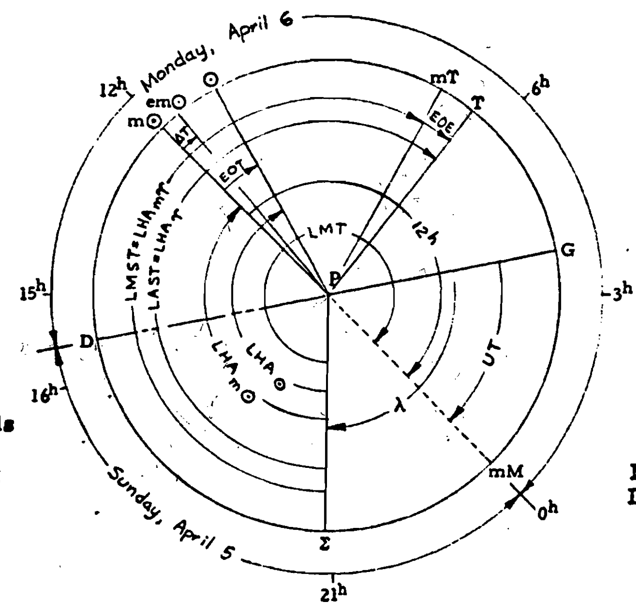


Fig. 4.6 Time diagram showing the relations between the various kinds of astronomical times. The angles corresponding to EOE, EOT, and ΔT are very much enlarged for clarity.

em⊙ = ephemeris mean sun
m⊙ = mean sun
⊙ = true sun
mT = mean equinox
T = true equinox
mM = mean midnight
EOT = equation of time
EOE = equation of the equinoxes
ΔT = ET-UT
LMST = local mean sidereal time
LAST = local apparent sidereal time
LMT = local mean time
LAT = local apparent time

4.11 Conversion of time scales. In solving certain of the problems which arise in Engineering Astronomy, it often becomes necessary to convert from one time scale to another. Among the time scales most frequently involved in such conversions are the following: local mean solar time, local mean sidereal time, zone time, Universal Time (UTC, UT1), and Ephemeris Time. A few of the more frequently required conversions are discussed in detail in the following section.

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4.11.1 To Change from Zone Time to Local Apparent Sidereal Time. To obtain the local apparent sidereal time on a given meridian corresponding to a specified instant of zone time, the following method may be used:

(a) Convert zone time to Coordinated Universal Time using eq. 4-27, UTC = Z.T. + Z.D.
(b) Convert UTC to UT1 using eq. 4-26, UT1 = UTC + ΔUT1.
(c) Enter the Almanac table of Universal and Sidereal Times with the above UT1 date as argument; take out the tabular value of Greenwich mean sidereal time at 0^hUT1 on that date.
(d) Multiply the value of UT1 from (b) by $1 + \sigma = 1.0027379093$
(e) Add the values from (c) and (d); the result is the Greenwich mean sidereal time at the desired epoch.
(f) Interpolate the tabular values of EOE to the epoch of UT1.
Add the values from (e) and (f); the result is the Greenwich apparent sidereal time at the desired epoch.
(h) Subtract the west longitude from the value found in (g); the result is the local apparent sidereal time.

Example: On 1983 March 28, in longitude 106°32'06"W, at $19^h27^m16^s$ zone 7 time, find the LAST.

Zone Time	1983 March $28^d19^h27^m16^s$
+ Zone Description	$+7^h$
(a) Coordinated Universal Time (UTC)	1983 March $29^d\ 2^h27^m16^s$
ΔUT1 (from U.S.N.O. bulletin)	$-0\overset{s}{.}0250$
(b) UT1	1983 March $29^d\ 2^h27^m15\overset{s}{.}9750$
(c) GMST @ 0^hUT1 on 3/29	$12^h23^m20\overset{s}{.}2477$
(d) $(1+\sigma)\cdot$(UT1) = GMST since 0^hUT1	$+2^h27^m40\overset{s}{.}1671$
(e) GMST @ epoch	$14^h51^m00\overset{s}{.}4148$
(f) EOE @ epoch (by interpolation)	$-1\overset{s}{.}0675$
(g) GAST @ epoch	$14^h50^m59\overset{s}{.}3473$
(h) $-\lambda_w$	-7 6 8.4
LAST	$7^h44^m50\overset{s}{.}9473$

4.11.2 To Change from Local Apparent Sidereal Time to Zone Time. To obtain the zone time on a specified meridian corresponding to a given instant of local apparent sidereal time, the following method may be used.

(a) Add the west longitude to the given local apparent sidereal time; the result is, by eq. 4-10, the Greenwich apparent sidereal time.
(b) Enter the Almanac table of Universal and Sidereal times with the given calendar date as argument; since the UT1 epoch is at first unknown, assume it to be zero, and take out the tabular value of EOE at 0^hUT1.
(c) Subtract the value found in (b) from that found in (a); the result is the Greenwich mean sidereal time.
(d) Again from the table, take out the Greenwich mean sidereal time at 0^hUT1 on the same date.
(e) Subtract the value found in (d) from that found in (c); the result is the change in Greenwich mean sidereal time since 0^hUT1.
(f) Divide the result found in (e) by $1 + \sigma = 1.0027379093$; the result is the change in UT1 since 0^hUT1, hence is UT1 itself.

Since the value of UT1 was not known in step (b), the value of EOE used was necessarily not the true value, but only a first approximation. Now that a trial value of UT1 has been determined, a second, and closer, approximation to EOE can be found by interpolation, and steps (c) thru (f) repeated to yield a better approximation to UT1, and so on until a desired level of accuracy is obtained. In practice, two interations will be found to suffice.

(g) Using the final value for UT1 as argument, determine the corresponding value of ΔUT1 = UT1-UTC from an appropriate source, such as the Series 7 bulletins of the U. S. Naval Observatory. The argument of ΔUT1 in these bulletins is strictly UTC, but in practice UT1 may be used as argument with no loss of accuracy. Subtract the value of ΔUT1 thus found from UT1; the result is UTC of the desired epoch.

(h) Subtract the zone description from the value of UTC found in (g); the result is the zone time of the desired epoch. Remember that the calendar date is a significant part of the UTC, and that it may be necessary to repeat the entire above process from step (b), if the wrong date was used at first.

Example: The local apparent sidereal time for an observer in longitude $106^{\circ}32'06''$W (zone description = + 7^h) is $7^h44^m50^s.9375$; the local date is 1983 March 28. Find the corresponding zone time.

	LAST	$7^h44^m50^s9473$	
	+ λ_w	+7 6 8.4	
(a)	~~GAST~~	$14^h50^m59^s.3473$	$59^s.3473$
(b)	-EOE (UT1 unknown at first; assume as zero)	−(−1.0581)	−(−$1^s.0590$)
(c)	GMST	$14^h51^m00^s.4054$	$60^s.4063$
(d)	−GMST @ 0^hUT1 on 1983 March 28^d	-12 19 23.6924	-23.6924
(e)	GMST	$2^h31^m36^s.7130$	36.7139
(f)	÷ $(1 + \sigma)$	$2^h31^m11^s.8750$	$11^s.8759$
(g)	− ΔUT1 from U.S.N.O. bulletin		−(−$^s.0251$)
	UTC	2^h31^m	$11^s.8010$
(h)	− Z.D.	-7^h	
	Z.T. 1983 March 27^d	19^h31^m	$11^s.8010$

but this is on the wrong day, ∴ must repeat from (d)

(a)	GAST	$14^h50^m59^s.3473$	$59^s.3473$
(b)	−EOE	−(−1.0666)	−(−1.0675)
(c)	GMST	$14^h51^m00^s.4139$	60.4148
(d)	−GMST @ 0^hUT1 on 1983 March 29^d	-12 23 20.2477	-20.2477
(e)	GMST	$2^h27^m40^s.1662$	$40^s.1671$
(f)	÷ $(1 + \sigma)$		
	UT1 1983 March 29^d	$2^h27^m15^s.9741$	$15^s.9750$
(g)	− ΔUT1 from U.S.N.O. bulletin		−(−$^s.0250$)
	UTC 1983 March 29^d	2^h27^m	$16^s.0000$
(h)	−Z.D.	-7^h	
	Z.T. 1983 March 28^d	19^h27^m	$16^s.0000$

4.11.3 To change from Local Mean Time to Zone Time. There are tables in both the Astronomical Almanac and the Nautical Almanac which involve values of local mean time; each observer must convert these values to his correct zone time. This conversion may be made by means of the following procedure:

(a) Obtain the UTC corresponding to the specified epoch of local mean time through the relation

$UTC = LMT + \lambda_w$ (Eq. 4-18, using UTC for UT)

(b) Obtain the corresponding zone time from the relation

$ZT = UTC - ZD$ (Eq. 4-27)

Combining the above steps, we may write

$$ZT = LMT + \lambda_w - ZD \qquad (4\text{-}28)$$

from which it is seen that, to obtain the zone time corresponding to a specified instant of local mean time, add to the LMT a correction of $(\lambda_w - ZD)$, that is, add a correction of 4^m for every degree by which the observer is west of his standard meridian; if he is east of his standard meridian, the correction is to be subtracted.

Example: The local mean time of a certain event, as observed from a place in $106^\circ\ 30'$ W longitude, is $5^h\ 16^m$; find the corresponding zone time.

The nearest standard meridian is 105° (see Table 4.1), so that the zone description is 7^h. The time equivalent of $106^\circ\ 30'$ is $7^h 6^m$; the correction from local mean time to zone time is thus 6^m, to be added, since the observer is west of his standard meridian. The zone time of the event is then

$$\begin{aligned} ZT &= LMT + (\lambda_w - ZD) \\ &= 5^h\ 16^m + 6^m \\ ZT &= 5^h\ 22^m. \end{aligned}$$

From the foregoing discussion, it is seen that to convert from zone time to local mean time, the inverse process may be used.

Exercises

4- 1. Using the formula for general precession from the Almanac 'Explanation' section, show that the period of the general precession is about 26,000 years.

4- 2. The local hour angle of the vernal equinox referred to a place in 160° W longitude is $14^h\ 15^m$; find the apparent sidereal time for the place.

4- 3. If the apparent sidereal time in longitude 320°W (40°E) at a given instant is known to be $6^h\ 14^m$, find the apparent sidereal time at each of the following longitudes:

(a) 20°W (b) 85°E (c) 160°W (d) 200°W

4- 4. If the equation of the equinoxes is $0^s.5$ at the instant referred to in problem 4-3, find the mean sidereal time on each of the meridians involved.

4- 5. Give the hour angle of the mean equinox at each of the meridians involved in problems 4-3 and 4-4.

4- 6. Find the mean sidereal time for a place in 67°E longitude at the instant of upper transit of Regulus, if the equation of the equinoxes is $-0^s.4$.

4- 7. If the hour angle of the mean sun is (a) 6^h (b) 17^h, what is the mean solar time?

4- 8. Using Fig. 4.2, find the local apparent time for part (a) of problem 4-7, for each of the following dates: (a) Feb. 20, (b) Sept. 3, and (c) Nov. 11.

4- 9. On each of the dates specified in problem 4-8, what will be the local mean time of apparent noon?

4-10. If the local mean time of a place in longitude 80°W is $4^h\ 20^m$, find the mean time of a place in longitude 120°W at that same instant.

4-11. Same as problem 4-10, except that the second place is in longitude 160°W.

4-12. The local mean time of a place in $108^{\circ}\ 30'\ 30''$ W longitude is February $3^d\ 4^h\ 10^m\ 20^s$. Find the Universal Time of this instant.

4-13. For each of the intervals of mean solar time listed below, find the corresponding interval of mean sidereal time, using the ratio given in section 4.5 of the text.

(a) $5^h\ 30^m$ (c) $22^h\ 18^m\ 36^s$

(b) $8^h\ 6^m$ (d) $15^h\ 42^m\ 22^s$

longer

EMOS Lat. 35° 3′ 6″ = 35.0516667
Lon. 106° 32′ 06″ W = 106.535

4-14. For each of the intervals of mean sidereal time listed below, find the corresponding interval of mean solar time, using the ratio given in section 4.5 of the text. smaller

(a) $4^h\ 24^m$ (c) $21^h\ 54^m\ 18^s$

(b) $9^h\ 15^m$ (d) $14^h\ 30^m\ 6^s$

4.7

4-15. Using the appropriate equation from section 4.6, calculate the Greenwich mean sidereal time for the epoch May $2^d\ 0^h$ UT1. Verify your answer by comparing it with the tabular value in the Almanac Table of Universal and Sidereal Times.

4-16. Find the ecliptic longitude and ecliptic latitude of the sun at the epoch October $7^d\ 14^h\ 26^m\ 13^s$ ~~E.T. (D.T.)~~ UTC

4-17. Find the right ascension and declination of Jupiter at the epoch June $8^d\ 3^h\ 20^m\ 42^s$ ~~E.T. (D.T.)~~ UTC page E27

4-18. Find the right ascension and declination of the moon at the epoch September $4^d\ 5^h\ 46^m\ 21^s$ ~~E.T. (D.T.)~~ UTC

4.8

4-19. By listening to radio station WWV (on base, dial 120), determine the current value of ΔUT1.

4-20. Using the above value of ΔUT1, convert the Coordinated Universal Time of $17^h\ 14^m\ 19^s.253$ to the corresponding epoch of UT1.

4-21. At a particular instant UTC = $3^h\ 48^m\ 26^s59$, ΔUT1 = $-0^s.28$, and ΔAT = 21^s. For that same instant, determine the corresponding values of TAI, UT1, and ET(DT).

For current values of ΔUT1 which may be required for the solution of any of the following exercises, ask the instructor.

4-22. Find the local mean and apparent sidereal times for Sacramento Peak Observatory, New Mexico, at the epoch February $6^d\ 20^h\ 10^m$ zone time.

4-23. Find the zone time of transit of the star Vega as seen from the Hale Observatory, Palomar Mountain, California, on June 4.

4-24. For the star and date specified by the instructor, find the zone time of transit of the star as viewed from EMOS. Sirius March 30^d

4-25. On the average, what is the daily difference in transit times for a given star and observing site? Be sure to specify whether it is earlier or later each day.

4-26. At what zone time on August 19 will the LAST be zero, as seen from the Mauna Kea Observatory, Hawaii?

April 23d 13h

4-27. For the epoch specified by the instructor, find ~~the~~ following quantities, for the sun: $31.^\circ 36488728$ $2^h 5^m 27.^s 57295$

(a) right ascension = ~~31.09032890 = 2h 4m 21.6790~~ $12.^\circ 71843960$ $12^\circ 43' 06.''4546$

(b) declination = ~~12.62155159 = 12° 37' 17.''5850~~

(c) true geocentric distance = ~~1.0056948~~

4-28. Give the sun's azimuth and altitude for April 21 at $12^h\ 00^m$ zone time as seen from EMOS.

4-29. Repeat the preceding exercise, but for Venus.

4-30. Give the zone times of sunrise, sunset, and the end of astronomical twilight for Mount Evans Observatory, Colorado, on the date of the summer solstice. Sect. A. LMT + difference of long. from std. meridian 4^m/degree

4-31. Give the zone times of moonrise and moonset for Catalina Observatory in Tucson, Arizona, on June 19.

$h_\gamma = h + \alpha$

$A = 223^\circ\ 23'\ 35''$

$a = 53^\circ\ 36'\ 31''$

$-2.^\circ 99307387 3$

+

−

CHAPTER 5

CORRECTIONS TO OBSERVATIONS

5.1 General Remarks. Engineering astronomy deals in general with the solution of the astronomical triangle based usually on observations made, or to be made, by means of portable field instruments such as a theodolite or an engineer's transit. In some cases, a larger device such as a telescope, radar, solar radiation collector, etc., may need to be pointed to some heavenly body at a specific time.

There are two different physical situations involved when making calculations to support such field operations, as follows:

(1) prediction of position -- the calculation of the obcentric (observer-centered) coordinates of the body at a specific epoch, either future or past, and

(2) reduction of observation -- the mathematical treatment of a completed observation so as to yield a desired result.

In either case, the effects of certain physical phenomena upon the body coordinates need to be considered and appropriate corrections applied as necessary.

The corrections to be discussed in this chapter are those due to the effects of refraction, semidiameter, geocentric parallax, diurnal aberration, and deflection of the vertical. A complete discussion of corrections due to systematic errors of the observing instrument itself is beyond the scope of this text; however, Appendix A does contain a few remarks about methods of eliminating certain of these systematic errors.

5.2 Refraction. When a ray of light passes through the atmosphere of the earth, the continual variation of the air density along the path of the ray causes a continual change in its direction of travel. In a general way, the ray is bent downward due to the refractive effect of the air through which it passes. If the atmosphere is horizontally stratified, that is, if it is composed of a number of layers each of different density but with a constant density in any single layer, then the bending of the ray path takes place entirely in a vertical plane. There is therefore no effect on the azimuth of an observed heavenly body, that is,

$$\Delta A_{AR} = 0 \tag{5-1}$$

The effect of refraction on the observed altitude of a heavenly body is shown in Fig. 5.1, which represents the earth with center C, an observer at O, a heavenly body at B, and a horizontally stratified atmosphere extending from the earth's surface to the dashed circle. In vacuum, a light ray from B would proceed to O along the straight line BAO, and the true zenith distance z_0 would be measured by the observer at O. In the presence of an atmosphere, this ray would be bent downward within the atmosphere to follow the path BAK, thus missing the observer entirely. There would exist, however, another ray leaving B along the line BA' which would be bent downward so as to reach point O at an apparent zenith distance z_o' ; to the observer, it would appear that B were located along the line OB'. This leads to the conclusion that due to refraction, all objects appear nearer the zenith, and thus higher above the horizon, than they actually are. The angular difference

$$\Delta z_{AR} = z_o - z_o' \tag{5-2}$$

(or $\Delta a_{AR} = a_o' - a_o$, which is the same thing)

is called the refraction correction; it must be added to a predicted obcentric altitude, or subtracted from an observed altitude, to yield the corresponding other value.

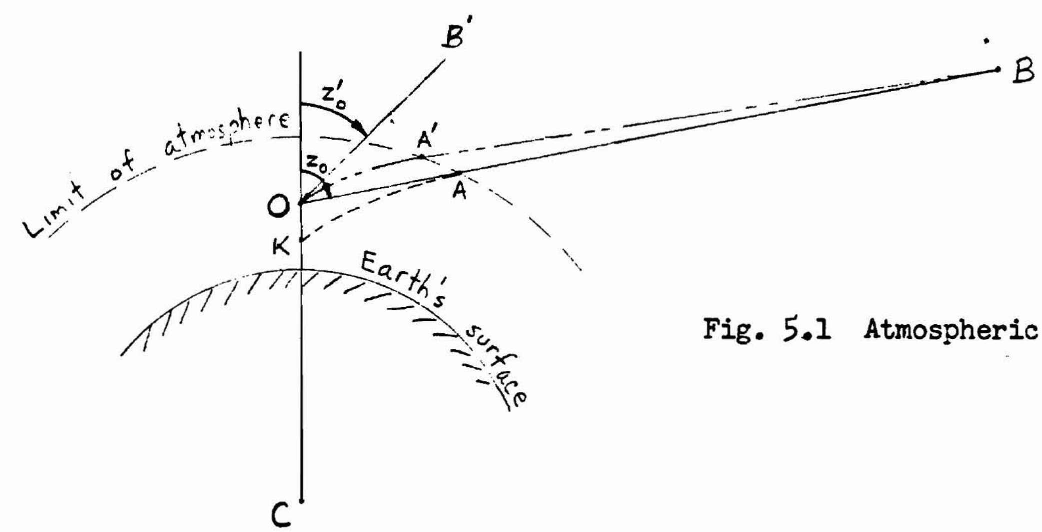

Fig. 5.1 Atmospheric refraction.

It should be noted that the refraction correction Δz_{AR} is not, in general, rigorously equal to the total bending of the light ray between points B and O; Fig. 5.2 may help make this clear. The ray from B which ultimately reaches the observer at O departs from B at an angle θ above the straight line BO. The total bending of the ray is the angle β between the tangents to the path at B and at O, whereas the refraction correction is strictly the angle Δz_{AR} formed at the observer by the straight line OB and the tangent to the incoming ray, i.e., the straight line OB'. From the geometry of the figure, it is seen that

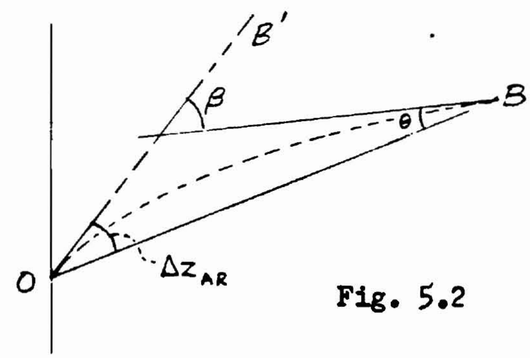

Fig. 5.2

$$\Delta z_{AR} = \beta - \theta \tag{5-3}$$

As the point B recedes to a very great distance from O, the angle θ approaches zero and Δz_{AR} approaches β ; in this limiting case, the total bending of the ray is exactly equal to the refraction correction, which is then called the astronomic refraction. It can be shown (see Appendix E) that all of the natural heavenly bodies are sufficiently far away so that θ is 1" or less, hence may usually be neglected for engineering purposes. For objects relatively near to the earth, such as artificial satellites, θ will be much larger than 1"; the treatment of refraction for such cases is not within the scope of this text.

Various formulas for computing the value of the astronomic refraction correction have been used in recent years. The simpler formulas are usually developed by neglecting the earth's curvature and assuming that all of the refraction takes place at the upper surface of the atmosphere; more elaborate derivations take the earth's curvature into account and also include the effects of barometric pressure, temperature, and relative humidity. Since the resulting analytical expressions are somewhat complex, numerical integration is frequently used to obtain a solution.

A defect in all analytical methods is that the actual ray path is a function of the state of the atmosphere at every point along the path, and this is never known in a practical situation. The analyst can only assume some atmospheric model, with arbitrary standard conditions of pressure, temperature, etc., at the observer, and realize that the result will represent, at best, an approximation to a statistical mean (i.e., average) refraction.

An alternate approach is to construct tables of refraction by means of repeated observations of stars from a known position; several such tables have been constructed and are still in use today, e.g., the Pulkovo tables and the Greenwich tables. Again, these tables necessarily yield statistical mean values of refraction, since the actual atmospheric state corresponding to a particular individual observation is never known all along the ray path.

5.2.1 Refraction correction to observed altitude.

The Nautical Almanac contains tables of astronomic refraction corrections which are based upon extensive theoretical and practical investigations by Garfinkel. The data in the tables are for assumed sea-level standard conditions of 1010 millibars and 10° Celsius.

By plotting and smoothing the N.A. tabular data, and by applying regression analysis methods to the results, the following two formulas have been obtained. Each formula represents, within its indicated range of validity, a match to the smoothed N.A. data within 1 arc-second for altitudes above 5° and within 3 arc-seconds for altitudes below 5°.

For the region $5^{\circ} \leq a' \leq 90^{\circ}$: *a' = observed altitude*

$$\overline{\Delta a}_{AR} = a' - a = \frac{58''\!.2}{\tan a'} - \frac{0''\!.058}{\tan^3 a'} + \frac{0''\!.000068}{\tan^5 a'} \qquad (5\text{-}4)$$

— @ ≈ 8°+

@ ≈ 20° becomes significant

For the region $0^{\circ} \leq a' \leq 5^{\circ}$:

$$\overline{\Delta a}_{AR} = a' - a = 1''\!.833\, a'^4 - 29''\!.23\, a'^3 + 196''\!.96\, a'^2 - 778''\!.9\, a' + 2070''\!.0 \qquad (5\text{-}5)$$

where $\overline{\Delta a}_{AR}$ = mean astronomic refraction

a' = observed altitude, degrees

a = true (corrected) altitude, degrees

Eqs. 5-4 and 5-5 give the mean astronomic refraction for the assumed standard conditions of 760 millimeters of mercury and 10° Celsius. Conversion to other conditions may be made by multiplying by appropriate factors as indicated:

1010 millibars = 760 mm Hg

$$\Delta a_{AR} = a' - a = \overline{\Delta a}_{AR}\, K_b\, K_t \qquad (5\text{-}6)$$

Where $K_b = \dfrac{b}{760}$

b = barometric pressure in mm of mercury

$K_t = \dfrac{283}{273 + C}$

C = atmospheric temperature in °Celsius

5.2.2 Refraction correction to predicted (true) altitude.

The preceding discussion is for the case in which an observed altitude is to be reduced to the true altitude by removing the effect of refraction. In the reverse case, when it is desired to convert from a predicted true altitude to the corresponding altitude including refraction, the following equations may be used, with errors as before:

For the region $5^\circ \leq a \leq 90^\circ$:

$$\overline{\Delta a}_{AR} = a' - a = \frac{58''.1}{\tan a} - \frac{0''.070}{\tan^3 a} + \frac{0''.000086}{\tan^5 a} \qquad (5\text{-}7)$$

For the region $-0^\circ 34' 34'' \leq a \leq 5^\circ$:

$$\overline{\Delta a}_{AR} = a' - a = 0''.711\ a^4 - 12''.79\ a^3 + 103''.4\ a^2 - 518''.2\ a + 1735''.0 \qquad (5\text{-}8)$$

Eqs. 5-7 and 5-8 give the mean astronomic refraction for the assumed standard conditions of 760 millimeters of mercury and 10° Celsius. Conversion to other conditions may be made by multiplying by appropriate factors as indicated:

$$\Delta a_{AR} = a' - a = \overline{\Delta a}_{AR}\ K_b\ K_t \qquad (5\text{-}9)$$

where K_b and K_t are defined as before.

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5.2.3 The effect of refraction on the right ascension, local hour angle, and declination of a body is shown in Fig. 5.3, which represents the celestial sphere with the observer's zenith at Z. Point P is the celestial north pole; B is the true position of a body and B' is its apparent position as affected by the refraction $r = \Delta a_{AR}$. The hour circles PBF and PB'F' are separated by the angular amount $\Delta h = h' - h$, the difference in the local hour angles of B' and B. The figure shows that Δh is also equal to $\Delta\alpha = \alpha - \alpha'$, the difference in the right ascensions of B and B'.

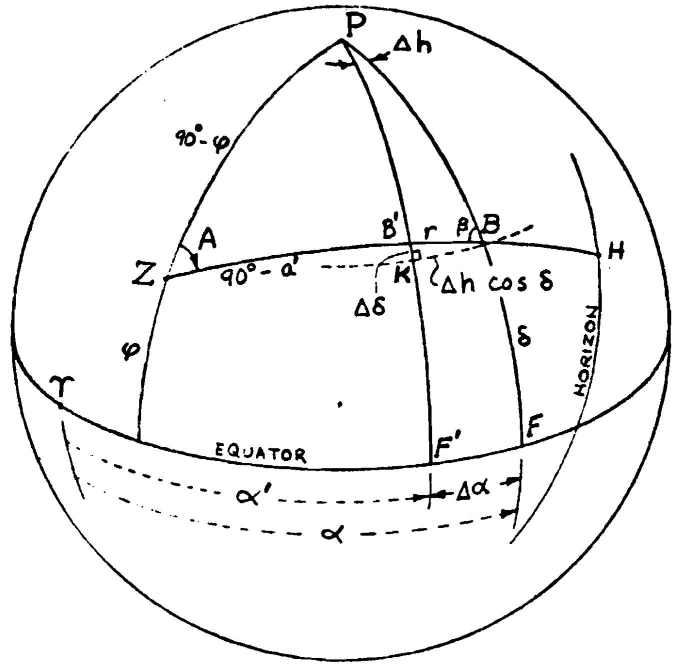


Fig. 5.3 The effect of refraction upon the right ascension, local hour angle, and declination of a body.

In the astronomic triangle PZB, the parallactic angle β at B is found from the four-consecutive-parts formula (Eq. 3-5) to be:

$$\tan\beta = \frac{\sin(360°-h)}{\dfrac{\sin(90°-\delta)}{\tan(90°-\varphi)} - \cos(90°-\delta)\cos(360°-h)} \qquad (5\text{-}10)$$

which, upon substitution of the appropriate co-functions, may be written as

$$\tan\beta = \frac{-\sin h}{\cos\delta\,\tan\varphi - \sin\delta\,\cos h} \qquad (5\text{-}11)$$

In the spherical triangle PBB', again using the four-consecutive-parts formula,

$$\tan \Delta h = \frac{\sin \beta}{\frac{\sin(90 - \delta)}{\tan r} - \cos(90^\circ - \delta)\cos \beta}$$

$$\tan \Delta h = \frac{\sin \beta}{\frac{\cos \delta}{\tan r} - \sin \delta \cos \beta} \qquad (5\text{-}12)$$

$$\Delta\alpha = \alpha - \alpha' = \Delta h \qquad \text{or} \qquad \alpha' = \alpha - \Delta h \qquad (5\text{-}13)$$

$$\Delta h = h' - h \qquad \text{or} \qquad h' = h + \Delta h \qquad (5\text{-}14)$$

Again in the spherical triangle PBB', using the law of cosines,

$$\cos(90^\circ - \delta') = \cos(90^\circ - \delta) \cos r + \sin(90^\circ - \delta) \sin r \cos \beta$$

$$\sin \delta' = \sin \delta \cos r + \cos \delta \sin r \cos \beta \qquad (5\text{-}15)$$

Equations 5-13, 14, and 15 then give the corrected right ascension, hour angle, and declination.

It should be noted that in Eq. 5-12, as the refraction r approaches zero, its tangent also approaches zero, and the first term in the denominator tends to infinity. This is mathematically correct, and leads to the correct result that Δh also approaches zero. When the solution is being made by means of a calculator or computer, proper care must be taken to assure of a correct value for Δh.

The above treatment is rigorous; in many cases, however, the approximate method discussed below will be sufficiently accurate.

The declination circle through B cuts the hour circle PB'F' at the point K forming the right triangular figure BKB'. The side KB' is equal to $\Delta\delta = \delta' - \delta$, the difference in declinations of B' and B; the hypotenuse BB' = r, the refraction; the side BK = $\Delta h \cos\delta$.

The figure BKB' is so small that it may, without serious error, be treated as a plane triangle as shown in Fig. 5-3a.

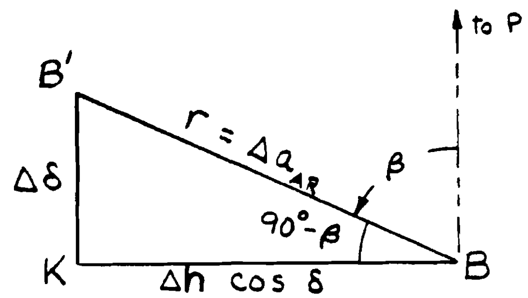


Fig. 5-3a

By inspection of the figure,

$$\Delta h \cos\delta = r\cos(90° - \beta) = r\sin\beta \qquad (5\text{-}16)$$

$$\Delta\delta = r\sin(90° - \beta) = r\cos\beta \qquad (5\text{-}17)$$

Then, since

$$\Delta h = \alpha - \alpha' = h' - h, \quad \text{and} \quad \Delta\delta = \delta' - \delta,$$

Eqs. 5-16 and 5-17 become

$$(\alpha - \alpha')\cos\delta = r\sin\beta$$

$$(h' - h)\cos\delta = r\sin\beta$$

$$\delta' - \delta = r\cos\beta$$

or, finally,

$$\alpha' = \alpha - \frac{r\sin\beta}{\cos\delta} \qquad (5\text{-}18)$$

$$h' = h + \frac{r\sin\beta}{\cos\delta} \qquad (5\text{-}19)$$

$$\delta' = \delta + r\cos\beta \qquad (5\text{-}20)$$

Eqs. 5-18, 19, and 20 then give the corrected right ascension, hour angle, and declination.

5.3 Parallax. In a general sense, the term "parallax" refers to the apparent angular shift in position of an object with respect to a remote background reference frame, when the object is viewed from two different locations. In astronomy, the background reference frame is composed of the most distant stars, which are so remote that they have no significant parallax of their own. The situation is shown in Fig. 5.4, in which B represents a body such as a star or a planet which appears to be at point B_1 when viewed from the observer at O_1 and at B_2 when viewed from the observer at O_2. The line O_2A is parallel to line O_1B_1; the parallax is measured by the angle AO_2B, which is seen to be equal to the angle O_1BO_2, hence the definition of parallax as the angle measured at the observed body between lines drawn to each of the observing locations.

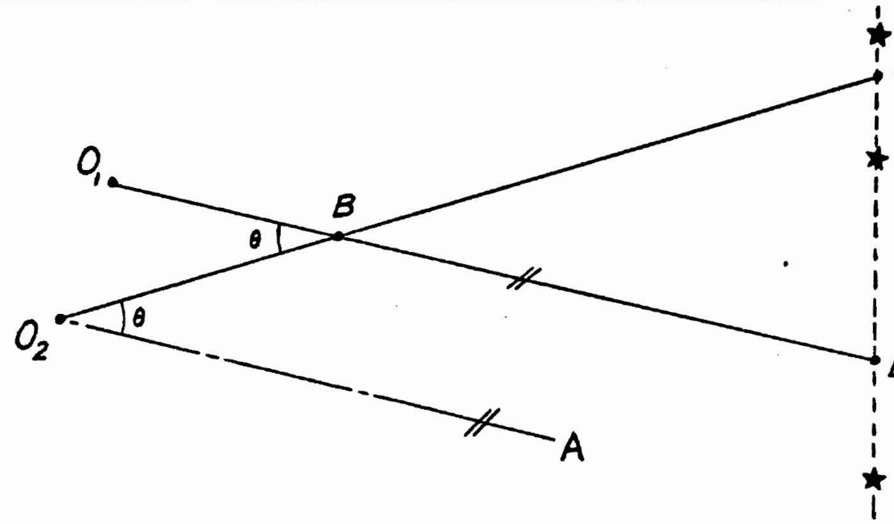


Fig. 5.4 The general case of astronomical parallax. The lines marked with a double slash are parallel.

There are two kinds of parallax in astronomy with which we are concerned; these are described in the paragraphs which follow. (correction to tabular data (not observed))

5.3.1 Annual parallax.

A star which is at a finite, even though very large, distance from the earth appears to trace out an elliptical path on the celestial sphere, as defined by the most remote of all the stars, in the course of the year. The paths for stars near the poles of the ecliptic are nearly circular; the paths of all the rest are foreshortened to appear as ellipses; for stars lying in the ecliptic plane, the paths are completely foreshortened into line segments along the ecliptic. The major axes of these elliptical paths are unaffected by this foreshortening, giving rise to the definition of annual (or stellar) parallax as the semimajor axis of the elliptical path traced out annually on the celestial sphere by the star. The annual parallax is also equal to the angle subtended at the star by the radius of the earth's orbit, as shown in Fig. 5.5 .

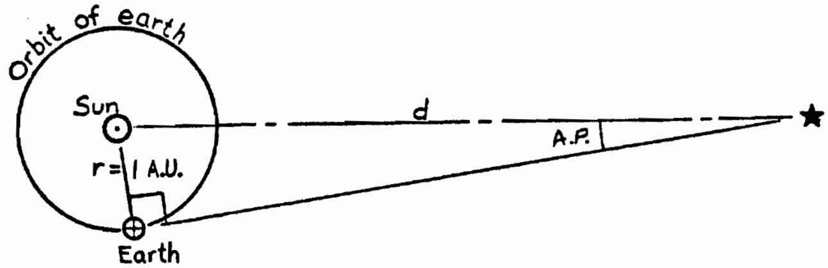


Fig. 5.5 Annual parallax.

The annual parallax of a star is related to the distance of the star from the sun by the relation

$$A.P.^{\circ} = \sin^{-1} \frac{1}{d} \qquad (5\text{-}21)$$

where A.P.° is the annual parallax in degrees and d is the distance from the sun to the star in astronomical units. Since the annual parallaxes of stars are very small, they are commonly given in seconds of arc, in which case the relation is written as

$$A.P.'' = \frac{648,000}{\pi} \cdot \frac{1}{d} \qquad (5\text{-}22)$$

where A.P." is the annual parallax in arc-seconds; the multiplier on the right-hand side is the number of arc-seconds per radian.

The annual parallaxes of stars are so small (the largest known value is less than 1 second of arc) that it may not be necessary to apply a specific correction for this purpose to any observation. Catalogs which give precise star positions do, however, take annual parallax into account in compiling their tabular data.

Table 5.1 lists a few stars of relatively large annual parallax.

Table 5.1. **Annual Parallaxes of Stars**

Star	Annual Parallax
Proxima Centauri	0".785 (largest known)
Barnard's Star	.54
61 Cygni	.29
Lacaille 9352	.27
Kapteyn's Star	.25
Cordoba 32416	.22
Groombridge 1830	.12

5.3.2 Geocentric parallax is the angle formed at the center of a heavenly body by two straight lines, one drawn to the center of the earth and the other drawn to the observer. It may also be regarded as the difference in direction of the body as viewed from the center of the earth and from the observer. A related quantity, the horizontal parallax, is defined as the angle formed at the center of a heavenly body by two straight lines, one drawn to the center of the earth and the other drawn tangent to an imaginary sphere which circumscribes the earth ellipsoid. The horizontal parallax may also be regarded as the angular semidiameter of the earth circumscribing sphere as viewed from the center of the heavenly body.

The above definitions are illustrated by Fig. 5.6 with the earth's center at C, a heavenly body at B, and an observer at O, located at a distance r_o from C. The earth circumscribing sphere is also shown, having a radius r_e equal to the equatorial radius of the earth ellipsoid. The geocentric parallax is the angle OBC, denoted on the figure by the letter p ; the horizontal parallax is the angle CBT, denoted on the figure by the letters HP. It is seen by inspection that the geocentric parallax p is equal to the difference $z_o - z_c$, and also to the difference $a_c - a_o$. In the figure and in the above definitions, no distinction is made between geodetic and geocentric latitudes, verticals, zenith distances, etc. As will be shown later in this chapter and in the Appendix, for all of the natural heavenly bodies other than the moon, a treatment based on assuming the earth as spherical will yield parallax correction values which are within less than one arc-second of the corresponding values derived from oblate earth considerations. The moon, because of its relative nearness to the earth, requires an approach which includes the effect of the earth's oblateness.

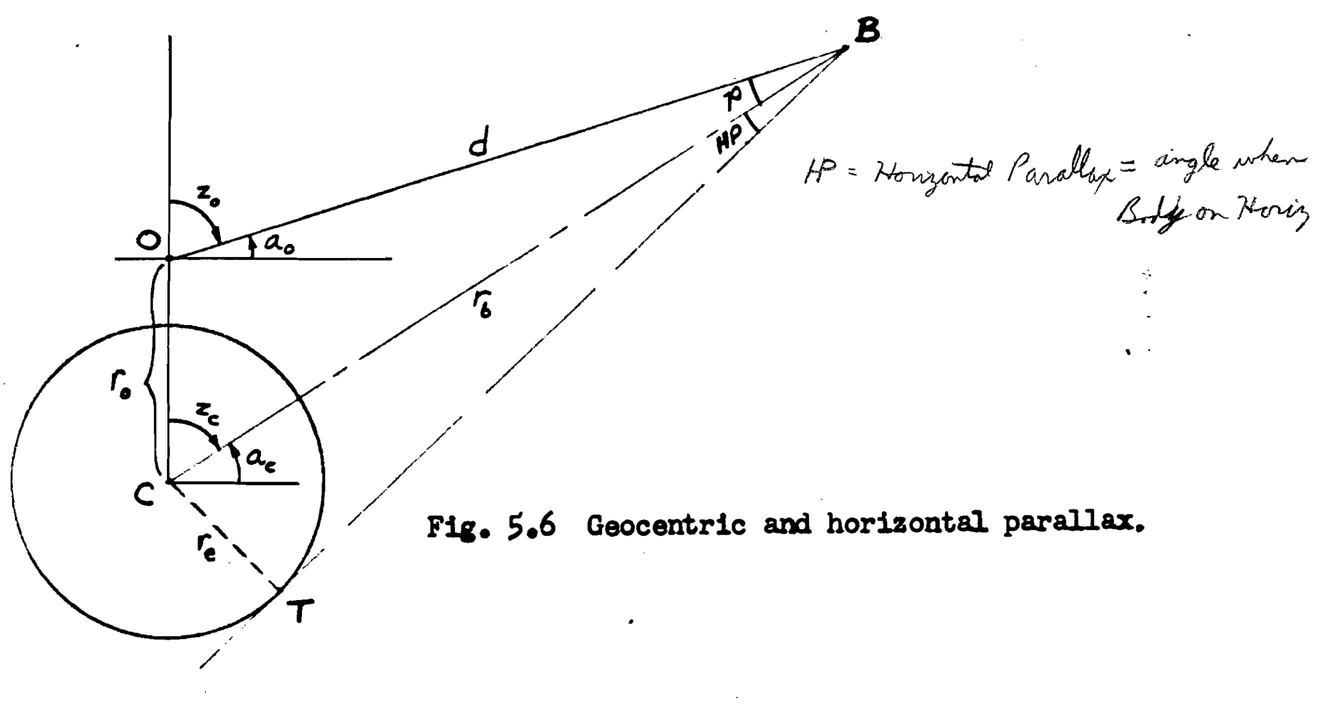


Fig. 5.6 Geocentric and horizontal parallax.

In the sections which follow, formulas for the calculation of the geocentric parallax will be derived. As explained earlier, there are two cases to be considered: the "prediction of position", in which the geocentric coordinates are known and the obcentric (observer-centered) coordinates to be found, and the "reduction of observation", in which the obcentric coordinates are known and the geocentric coordinates are to be found.

Values of the horizontal parallax, which may be desired for solution of the formulas for geocentric parallax, are given either explicitly or implicitly in the Astronomical Almanac. For the Sun, values are tabulated in Section C, at 0^h Dynamical Time daily. For the Moon, values are given in two places in Section D, once daily in sexagesimal units, at 0^h DT, and in decimal degree units in the polynomial form.

Horizontal parallax values for the planets are not tabulated; they may, however, be obtained by dividing the adopted value of equatorial horizontal parallax at unit distance ($8''.794$) by the true geocentric distance of the planet in Astronomical Units, values of which are given in Section E of the Almanac, at 0^h DT. daily.

Table 5.3, below, gives maximum values of the horizontal parallax for the various heavenly bodies, based on their mean nearest distance from the earth.

Table 5.3 Maximum horizontal parallaxes

Body	d_{min} (A.U.)	$HP_{max} = \frac{8''.794}{d_{min}}$
* Sun	1	$8''.794$
* Moon	.00257	3422.61
Mercury	.61	14.42
Venus	.28	31.41
Mars	.52	16.91
Jupiter	4.20	2.09
Saturn	8.53	1.03
Uranus	18.2	0.48
Neptune	29.0	.30
Pluto	38.5	.23
Stars	$\gg 1$	0

* adopted value (not minimum)

5.3.3 Geocentric parallax of bodies other than the Moon. Because of the assumption of a spherical earth, there is no effect on the observed azimuth, that is,

$$\Delta A_{GP} = 0 \qquad (5\text{-}23)$$

The effect of the geocentric parallax on the observed altitude is, by inspection of Fig. 5.6,

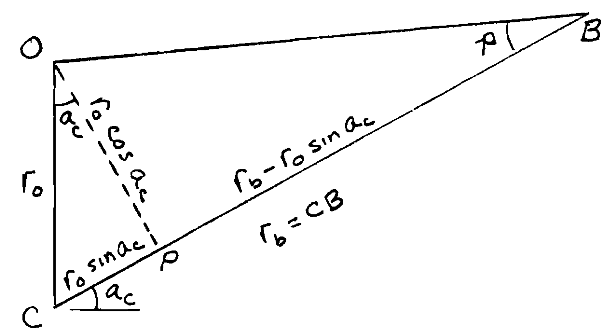
$$\Delta a_{GP} = a_c - a_o = z_o - z_c = p \qquad (5\text{-}24)$$

Case 1: Prediction of position. (A_c, a_c given; A_o, a_o required)

Referring again to Fig. 5.6, in the triangle OCB, drop a perpendicular from O to point P on side BC. By inspection of the resulting figure, shown here for convenience,

$\angle COP = a_c \qquad OP = r_o \cos a_c$

$CP = r_o \sin a_c \qquad PB = r_b - r_o \sin a_c$



Then $\tan p = \dfrac{OP}{PB} = \dfrac{r_o \cos a_c}{r_b - r_o \sin a_c}$

which, after dividing thru by r_o, becomes

$$\tan p = \frac{\cos a_c}{r_b/r_o - \sin a_c} \qquad (5\text{-}25)$$

Since the horizontal parallax is related to the geocentric distance via $\sin HP = \dfrac{r_e}{r_b}$ (as may be seen from Fig. 5.6), then eq. 5-25 may be written in the form

$$\tan p = \frac{\cos a_c}{\dfrac{r_e}{r_o \sin HP} - \sin a_c} \qquad (5\text{-}26)$$

When the observer is on the earth circumscribing sphere, $r_o = r_e$, and the above two numbered expressions simplify to

$$\tan p = \frac{\cos a_c}{r_b - \sin a_c} \qquad (5\text{-}25a)$$

$$\tan p = \frac{\cos a_c}{\dfrac{1}{\sin HP} - \sin a_c} \qquad (5\text{-}26a)$$

Having thus found the geocentric parallax p, the observed altitude is found from Eq. 5-24, put in the form

$$a_o = a_c - p \qquad (5\text{-}24a)$$

5.3.3 cont'd-- Geocentric parallax of bodies other than the Moon.

Case 2: Reduction of observation. (A_o, a_o given; A_c, a_c required)

From triangle OCB in Fig. 5.6,

$$\frac{\sin p}{r_o} = \frac{\sin(90° + a_o)}{r_b}$$

Since $\sin(90° + a_o) = \cos a_o$, the above expression may be written as

$$\sin p = \frac{r_o}{r_b} \cos a_o \qquad (5\text{-}27)$$

By inspection of the figure, $r_b = \frac{r_e}{\sin HP}$, so that

$$\sin p = \frac{r_o \cos a_o}{r_e / \sin HP}, \text{ or}$$

$$\sin p = \frac{r_o}{r_e} \sin HP \cos a_o \qquad (5\text{-}28)$$

For an observer on the surface of the earth circumscribing sphere, $r_o = r_e$, and the above expression simplifies to

$$\sin p = \sin HP \cos a_o \qquad (5\text{-}29)$$

Having thus found the geocentric parallax p, the geocentric altitude is found from Eq. 5-24, put in the form

$$a_c = a_o + p \qquad (5\text{-}24b)$$

5.4 Semidiameter. The angle subtended at the center of the earth by the linear radius of a heavenly body is known as the geocentric angular semidiameter, or simply as the semidiameter, of the body.

Referring to Fig. 5.8, if r represents the distance from the center of the earth to the center of a heavenly body at B and R_b represents the linear radius of the body, then the angular semidiameter s of the body is given by

$$\sin s = \frac{R_b}{r}, \text{ or}$$

$$s = \sin^{-1} \frac{R_b}{r} \qquad (5\text{-}4.1)$$

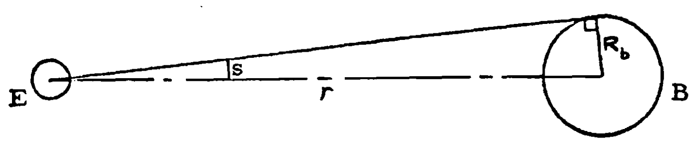
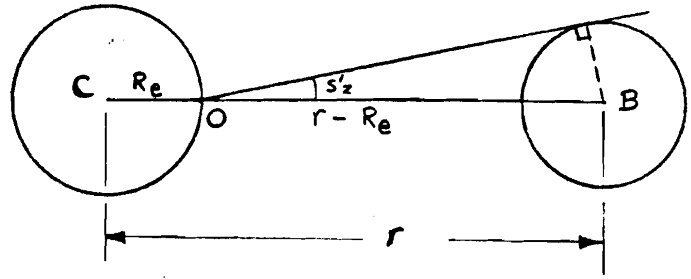


Fig. 5.8 Relation between the angular semidiameter s of a body, its linear radius R_b, and its distance from the center of the earth.

The Ephemeris gives the semidiameter of the sun and planets at 0^h ET each day; the semidiameter of the moon is given twice daily, at 0^h and 12^h ET. The adopted value for the semidiameter of the sun at unit distance of 1 A.U. is 16' 01".18 ; that for the moon is 15' 32".58 at a unit distance of 60.2682 equatorial radii of the earth.

Since in making observations of the sun or moon it is difficult to sight with accuracy at the center, it is common practice to sight on the limb, or edge, instead. The limb is well defined and the setting can be made with precision. The position of the center is obtained by correcting the observation for the semidiameter. The amount of the correction is different for different altitudes, because the body is at different distances from the observer, as shown in Fig. 5.9 . The apparent enlargement of the semidiameter as the body's altitude increases is known as the augmentation of the semidiameter. In practice, the effect is of consequence only for the moon, as will be shown by the following analysis.

When a heavenly body is at the zenith of an observer, as shown in the figure at right, the augmented semidiameter is given by

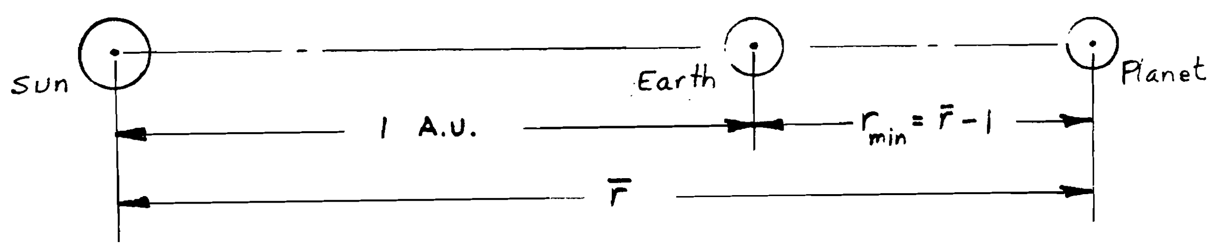


$$\sin s'_z = \frac{R_b}{r - R_e}$$

The augmentation at the zenith is

$$\Delta s_z = s'_z - s$$

where s is the geocentric semidiameter from Eq. 5-4.1 . The maximum zenithal augmentation occurs when the body is at minimum distance from the earth. In the case of the sun and moon, since the orbits involved are nearly circular, a mean value of r may be used to obtain an approximate value of Δs_{max} for each body. In the case of the planets, r is a minimum when the planet concerned is "lined up" with the earth and sun as indicated in the figure below. In the figure, $\bar{r}$ represents the mean orbital distance of the planet from the sun; orbital eccentricities and inclinations are ignored.



The actual linear radius R_b of the planet need not be explicitly known, but may be replaced in the first expression on this page by its equivalent

$$R_b = r_1 \sin s_1$$

where r_1 = adopted unit distance for the body, and
s_1 = adopted semidiameter at unit distance .

The Ephemeris gives, in the Explanation section, the adopted semidiameters for the sun, moon, and planets at unit distance (1 A.U. for the sun and planets, 60.2682 equatorial earth radii for the moon). From those values and from the formulas and considerations above, together with necessary orbital data from Table 1.1, Table 5.2 may be constructed. Inspection of the last column of the table shows that, to a precision of 0."1 , the augmentation is negligible for all of the natural heavenly bodies except the moon.

Table 5.2 Semidiameters and maximum augmentations

Body	s_1	s_{max}	s'_{max}	Δs_{max}
Sun	961".18	961".18	961".22	0".04
Mercury	3.34	5.48	5.48	0
Venus	8.41	30.04	30.04	0
Moon	932.58	932.58	948.32	15.74
Mars	4.68	9.00	9.00	0
Jupiter	98.47	23.45	23.45	0
Saturn	83.33	9.77	9.77	0
Uranus	34.28	1.88	1.88	0
Neptune	36.56	1.26	1.26	0
Pluto	(not given)			

(Note: The values given above for Jupiter and Saturn are the equatorial ones.)

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5.4.1 Augmentation of the semidiameter of the Moon: rigorous method.

Fig. 5.9 illustrates a number of related quantities which are useful for the present development; among them are the geocentric semidiameter s , the obcentric semidiameter $s' = s + \Delta s$ (Δs is the augmentation), and the horizontal parallax HP. By inspection of the figure, it is seen that

$$R_b = r \sin s = d \sin s' \qquad (5\text{-}4.1)$$

Also by inspection,

$$r = R_e / \sin HP \qquad (5\text{-}4.2)$$

Combining the foregoing and rearranging produces

$$\sin s' = \frac{R_e \sin s}{d \sin HP} \qquad (5\text{-}4.3)$$

The angle a'_c is the geocentric altitude of the moon measured with respect to the geocentric vertical of the observer at O, and may be obtained from Eq. 5-33, repeated here for convenience,

$$\sin a'_c = \cos \Delta\varphi \sin a_c - \sin \Delta\varphi \cos a_c \cos A_c \qquad (5\text{-}33)$$

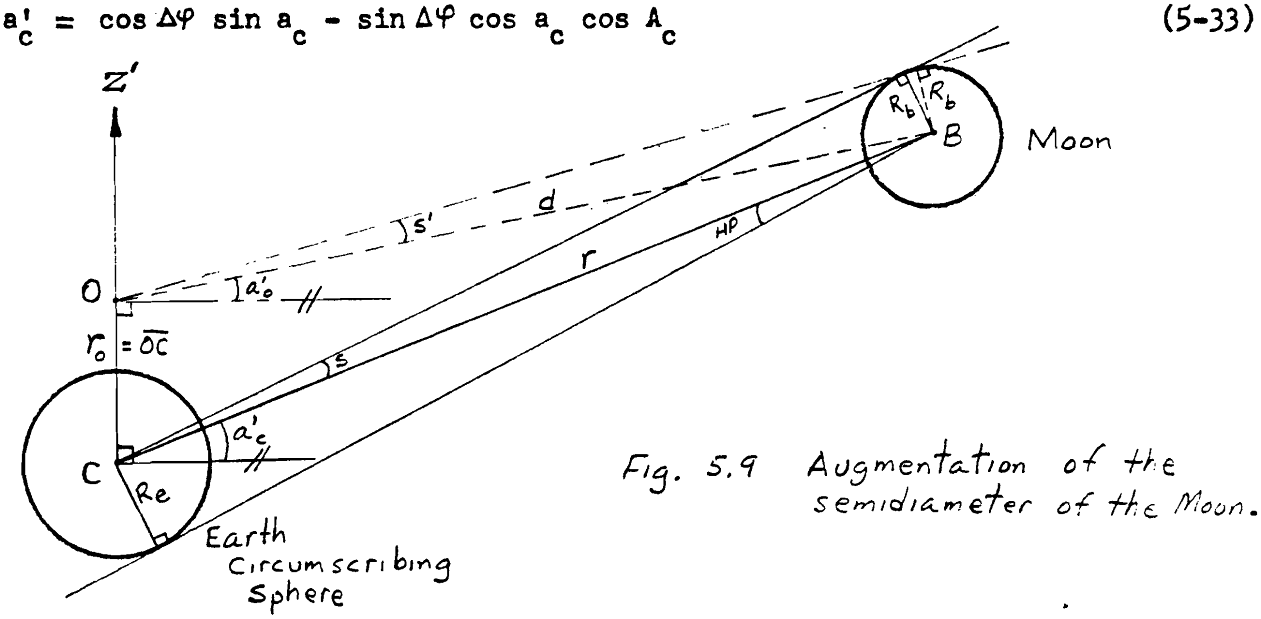

Fig. 5.9 Augmentation of the semidiameter of the Moon.

In triangle OCB, using the law of cosines,

$$d^2 = r^2 + r_o^2 - 2\, r\, r_o \cos(90° - a'_c) = r^2 + r_o^2 - 2\, r\, r_o \sin a'_c$$

Using eq. 4.2 in this expression yields

$$d^2 = \frac{R_e^2}{\sin^2 HP} + \frac{R_e^2\, r_o^2}{R_e^2} - \frac{2\, R_e}{\sin HP} \cdot \frac{R_e}{R_e} \cdot \frac{r_o \sin a'_c}{1}$$

$$d^2 = R_e^2 \left(\frac{r_o^2}{R_e^2} + \frac{1}{\sin^2 HP} - \frac{r_o}{R_e} \cdot \frac{2 \sin a'_c}{\sin HP} \right)$$

Taking the square root gives

$$d = R_e\sqrt{\left(\frac{r_o}{R_e}\right)^2 + \frac{1}{\sin^2 HP} - \frac{r_o}{R_e}\cdot\frac{2\sin a'_c}{\sin HP}} \tag{5-4.4}$$

Substituting the above expression for d into eq. 4.3 gives

$$\sin s' = \frac{R_e \frac{\sin s}{\sin HP}}{R_e\sqrt{\left(\frac{r_o}{R_e}\right)^2 + \frac{1}{\sin^2 HP} - \frac{r_o}{R_e}\cdot\frac{2\sin a'_c}{\sin HP}}}$$

which becomes, after a little rearrangement,

$$\sin s' = \frac{\sin s}{\sqrt{1 + \left(\frac{r_o}{R_e}\right)^2 \sin^2 HP - 2\left(\frac{r_o}{R_e}\right)\sin HP \sin a'_c}} \tag{5-4.5}$$

The preceding expression gives the augmented semidiameter s' as a function of the geocentric altitude a'_c referred to the geocentric horizon of the observer, thus is strictly intended for the "prediction of position" case. The corresponding expression for the "reduction of observation" case may be obtained in a similar fashion, using the relation

$$r^2 = r_o^2 + d^2 - 2\, d\, r_o \cos(90° - a'_o)$$

from Fig. 5.9, to obtain

$$\sin s' = \frac{\sin s}{\sqrt{1 - \left(\frac{r_o}{R_e}\right)^2 \sin^2 HP \cos^2 a'_o} - \frac{r_o}{R_e}\sin HP \sin a'_o} \tag{5-4.6}$$

in which a'_o is the obcentric altitude of the moon referred to the geocentric vertical and horizon of O.

5.4.2 Augmentation of the semidiameter of the Moon: approximate method.

The expressions for the augmented semidiameter of the moon derived in the preceding section are rigorous; analysis shows (see Appendix E) that the following approximate method may be used, with error less than about $0.''25$, in all cases where the observer is within 40 kilometers of the earth circumscribing sphere.

In Eq. 5-4.5, let r_o equal R_e, and a'_c equal 90°; also replace the sine of the semidiameters by the semidiameters in radians, since they are always small. The resulting expression is the maximum augmented semidiameter

$$s'_{max} = \frac{s}{1 - \sin HP}$$

Now replace the sine of the horizontal parallax by the equivalent expression $K \cdot HP$, where K is equal to the number of radians per arc-second, to obtain

$$s'_{max} = \frac{s}{1 - K \cdot HP}$$

where s, s', and HP are in arc-seconds

$$K = \frac{\pi}{180(3600)}$$

The maximum augmentation is then

$$\Delta s_{max} = s'_{max} - s = \frac{s}{1 - K \cdot HP} - s = s\left(\frac{1}{1 - K\,HP} - 1\right)$$

$$\Delta s_{max} = \frac{s \cdot K \cdot HP}{1 - K \cdot HP} = \frac{s}{\frac{1}{K \cdot HP} - 1}$$

The Ephemeris gives the relation between the moon's semidiameter and horizontal parallax as

$$s = 0.''0799 + 0.272453\ HP$$

where both s and HP are in arc-seconds.

Ignoring the constant term, the relation simplifies to

$$s = 0.272453\ HP \qquad \text{or} \qquad HP = \frac{s}{0.272453}$$

Using this relation, the expression for the maximum augmentation becomes

$$\Delta s_{max} = \frac{s}{\frac{180(3600)0.272453}{\pi s} - 1} = \frac{s}{\frac{56205}{s} - 1} \qquad (5\text{-}4.7)$$

The augmentation at other altitudes is given by

$$\Delta s = \frac{s}{\frac{56205}{s} - 1} \sin a \qquad (5\text{-}4.8)$$

As shown by Appendix E, the altitude a in the above expression may be either the geocentric or the obcentric altitude of the moon's center or of either limb (upper or lower)

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5.4.3 Correction to azimuth and altitude for the effect of semidiameter.

A portion of the celestial sphere is shown in Fig. 5.10 with the observer's zenith at Z; the solid circle represents the obcentric position of a body in vacuum, that is, if there were no atmosphere. Point C is the body center and the arc ZC is the vertical circle through the body. The vertical circle ZF_{LL} represents the circle swept by a theodolite whose vertical cross-wire is tangent to the left limb of the body at T. The semidiameter, increased by the augmentation in the case of the moon, is denoted by s . The difference in azimuths of the body's center and left limb is ΔA_{SD}; ${}_oa_c$ is the unrefracted obcentric altitude of the center and ${}_oa'_c$ is the corresponding refracted altitude; r is the refraction of the center.

In the spherical triangle ZCT,

$$\frac{\sin \Delta A_{SD}}{\sin s} = \frac{\sin 90°}{\sin(90° - {}_oa_c)}$$

$$\sin \Delta A_{SD} = \frac{\sin s}{\cos {}_oa_c}$$

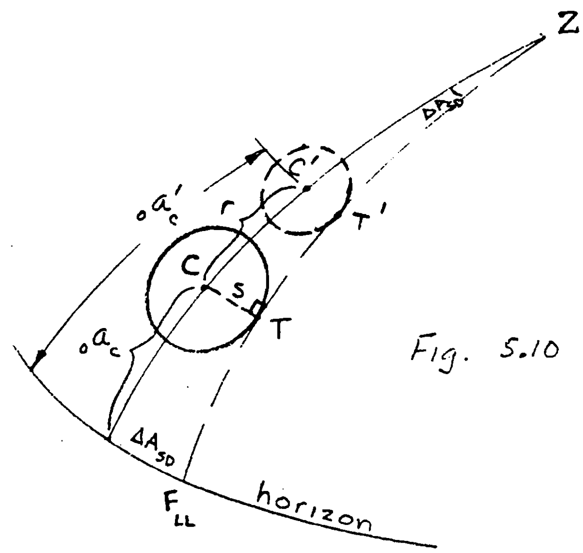


Fig. 5.10

Since the semidiameters are always small, the sine of the semidiameters may be replaced by the radian equivalents to give

$$\Delta A_{SD} = \frac{s}{\cos {}_oa_c} \qquad (5\text{-}4.9)$$

The obcentric altitude of the center, ${}_oa_c$, required for the solution of the above equation, is obtained either by calculation (including the geocentric parallax) or, in the case of reduction of observation, by observing the altitude of a limb or the center with as little loss of time as possible after the azimuth observation, and removing the effect of refraction.

The semidiameter correction to the altitude is simply the semidiameter itself,

$$\Delta a_{SD} = s \qquad (5\text{-}4.10)$$

The corrected obcentric azimuth and altitude are then

$$\left.\begin{aligned} A_{RL} &= A_c + \Delta A_{SD} \\ A_{LL} &= A_c - \Delta A_{SD} \\ a_{UL} &= {}_oa_c + s \\ a_{LL} &= {}_oa_c - s \end{aligned}\right\} \qquad (5\text{-}4.11)$$

near pole or near zenith

5.5 Aberration. Because the velocity of light is finite, the apparent direction of a moving celestial body as seen by a moving observer is not the same as the geometric direction of the body from the observer at that instant. This displacement of the apparent position from the geometric position, known as planetary aberration, may be attributed in part to the motion of the body and in part to the motion of the observer, these motions being referred to an inertial frame of reference. The first part, due to the motion of the body in the inertial frame of reference during the interval while light is propagating from the body to the observer, is known as the correction for light-time; this correction is included in the apparent positions for the sun, moon, and planets as given in the Almanac. The second part, due to the motion of the observer in the inertial frame of reference, is called the stellar aberration, since for the stars the correction for light-time is, of necessity, ignored.

The motion of an observer on earth is the resultant of the diurnal rotation of the earth on its axis, the orbital motion of the earth about the center of mass of the solar system (which for our purposes may be considered to be the same as the center of the sun), and the motion of this center of mass in space. The stellar aberration is thus made up of three components, known respectively as the diurnal aberration, the annual aberration, and the secular aberration. The secular aberration is practically constant for each star and hence may be ignored, leaving only the diurnal and annual aberrations to be considered.

In Fig. 5.11, let O_1 and B_1 represent the true positions of an observer and a celestial body, respectively, at some epoch I_1. At this instant, a ray of light leaves the body at B_1 and propagates at speed c towards point O_2, the position occupied by the observer at the epoch $I_2 = I_1 + \tau$, where τ is the time required for the light ray to reach point O_2. The instantaneous velocity of the observer at point O_2 is denoted by v, and is directed along the line $O_1'O_2M$. Point O_1' is located at a distance $d = v\tau$ from O_2.

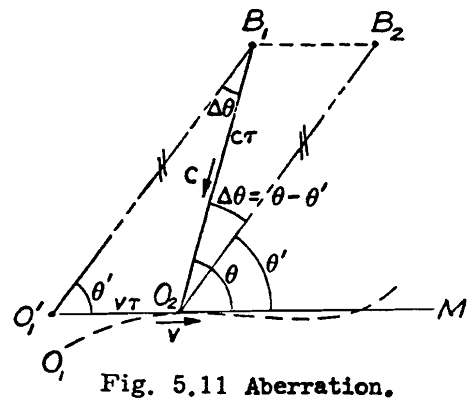


Fig. 5.11 Aberration.

The law of aberration is that the apparent direction of the body from the observer at this second epoch is measured by the angle θ' in the figure, whereas the true (geometric) direction is measured by the angle θ. Point B_2 then represents the apparent position of the body at the second epoch.

In triangle $O_2O_1'B_1$, we see that

$$\frac{v\tau}{\sin\Delta\theta} = \frac{c\tau}{\sin\theta'}$$

which becomes, upon eliminating τ and rearranging,

$$\sin\Delta\theta = \frac{v}{c}\sin\theta'$$

Since the angle $\Delta\theta$ is always small, we may write this expression as

$$\Delta\theta = \frac{v}{c}\sin\theta'$$

where $\Delta\theta$ is in radians; if $\Delta\theta$ is to be given in seconds of arc, it is necessary to multiply the right-hand side by the number of arc-seconds per radian, $\frac{648{,}000}{\pi}$, to give

$$\Delta\theta = \frac{648{,}000}{\pi}\frac{v}{c}\sin\theta' = \kappa\sin\theta'$$

(or $\Delta\theta = \kappa\sin\theta$

with no significant loss of accuracy, since θ and θ' are very nearly equal)

letting

$$\kappa = \frac{648{,}000}{\pi}\cdot\frac{v}{c} \qquad (5\text{-}36)$$

where κ, expressed in arc-seconds, is known as the <u>constant of aberration.</u>

When the earth's orbital speed is used in Eq. 5-36, the resulting value is κ_a, the <u>constant of annual aberration.</u> Taking the mean radius of the earth's orbit as 149,600,000 km (= 1 Astronomical Unit) and the length of the tropical year as $365\overset{d}{.}24219$, the mean speed of the earth in its orbit is

$$v_{\oplus} = \frac{2\pi r}{P} = \frac{2\pi(149{,}600{,}000)}{365.24219(86{,}400)} = 29.78631 \text{ km/s.}$$

The speed of light is given in the Almanac, to sufficient accuracy, as

$$c = 299{,}792.5 \text{ km/s .}$$

Using these values in Eq. 5-36, the constant of annual aberration is

$$\kappa_a = \frac{648,000}{\pi} \cdot \frac{29.78631}{299,792.5} = 20''.494 \tag{5-37}$$

(The Almanac value is 20".496.)

The effect of annual aberration is treated as a correction to the tabular position of a star rather than as a correction to an observation. The methods commonly used to obtain the numerical value of the correction are discussed in Chapter 6.

When the rotational speed of an observer on the surface of the earth is used in Eq. 5-36, the resulting value is κ_d, the constant of diurnal aberration. Letting ρ represent the geocentric radius of the observer in units of the earth's equatorial radius (= 6,378.140 km) and φ' = his geocentric latitude, and taking the length of the sidereal day from the Almanac as $23^h\ 56^m\ 04^s.09054$ (= $86,164^s.09054$) of mean solar time, the linear velocity of the observer towards the east is

$$v_o = \frac{2\pi r}{P} = \frac{2\pi\, 6378.140\, \rho \cos\varphi'}{86,164.09054}$$

$$v_o = 0.4651028\ \rho \cos\varphi' \ \text{km/s}$$

Again using Eq. 5-36, we obtain the constant of diurnal aberration as

$$\kappa_d = \frac{648,000}{\pi} \cdot \frac{0.4651028}{299,792.5}\, \rho \cos\varphi' = 0''.320\ \rho \cos\varphi' \tag{5-38}$$

or, upon dividing by 15,

$$\kappa_d = 0^s.0213\ \rho \cos\varphi' . \tag{5-39}$$

5.5.1 The diurnal aberration as given by Eqs. 5-38 and 5-39 may be resolved into corrections to both right ascension and declination, as shown by the following analysis.

In Fig. 5.12, the celestial sphere is shown with the observer at O, his zenith at Z, his east point at E, and a star at B. The component of the observer's instantaneous velocity vector which is caused by the diurnal rotation of the earth is directed along the line OE, causing the apparent position of the star at B to be shifted to B'. The amount of the shift is given by $\kappa_d \sin\theta$; it is so small that the figure BQB' may, without significant loss of accuracy, be treated as a plane triangle.

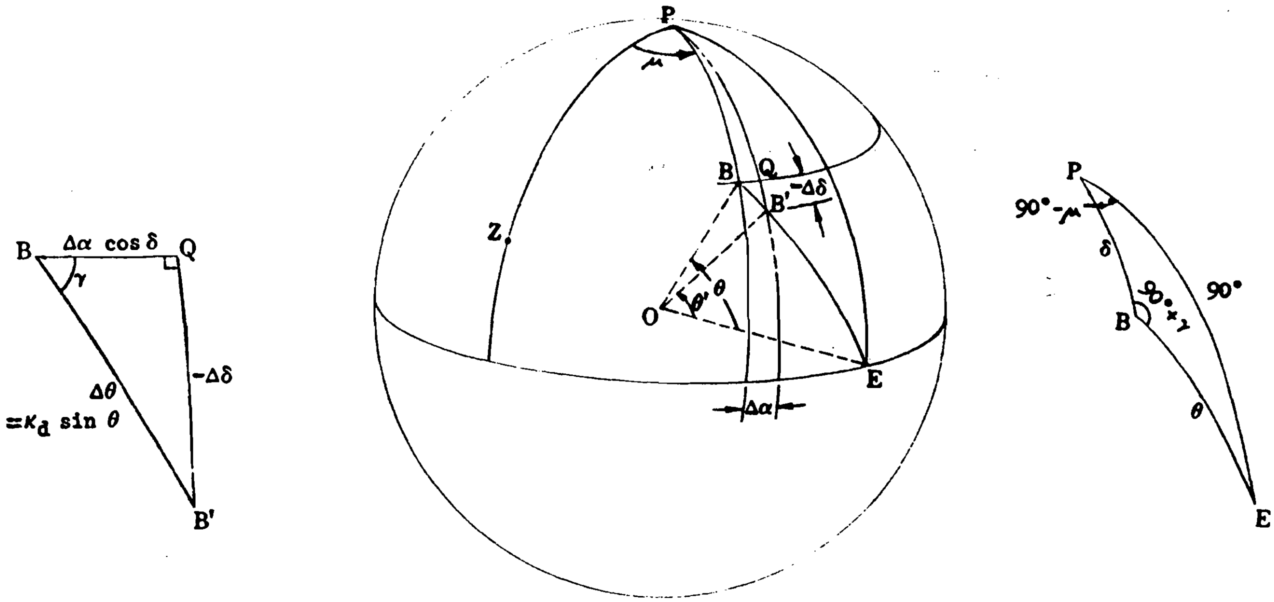


Fig. 5.12 The effect of diurnal aberration on right ascension and declination.

By inspection,

$$BQ = \Delta\alpha \cos\delta = \Delta\theta \cos\gamma$$

$$\Delta\alpha \cos\delta = \kappa_d \sin\theta \cos\gamma$$

$$\Delta\alpha = \kappa_d \sec\delta \sin\theta \cos\gamma \tag{5-40}$$

and

$$-\Delta\delta = \kappa_d \sin\theta \sin\gamma \tag{5-41}$$

In the spherical triangle PBE, using the law of sines (Eq. 3-3), we have that

$$\frac{\sin\theta}{\sin(90^\circ-\mu)} = \frac{\sin 90^\circ}{\sin(90^\circ+\gamma)}$$

$$\sin\theta \cos\gamma = \cos\mu \tag{5-42}$$

Again, using the five-parts formula (Eq. 3-2), we obtain

$$\sin\theta \cos(90^\circ+\gamma) = \cos 90^\circ \sin(90^\circ-\delta) - \sin 90^\circ \cos(90^\circ-\delta) \sin\mu$$

$$\sin\theta \sin\gamma = \sin\delta \sin\mu \tag{5-43}$$

Substituting Eqs. 5-42 and 5-43 into Eqs. 5-40 and 5-41 respectively, and using the relation between meridian angle μ and hour angle h as discussed in section 3.5, we get

$$\Delta\alpha = \kappa_d \sec \delta \cos h$$

$$\Delta\alpha_{DA} = \alpha' - \alpha = 0^s.0213 \rho \cos \varphi' \cos h / \cos \delta \quad (5\text{-}44)$$

$$\Delta\delta_{DA} = \delta' - \delta = 0''.320 \rho \cos \varphi' \sin h \sin \delta \quad (5\text{-}45)$$

For a star on the meridian, h is either 0^h (upper transit) or 12^h (lower transit) so that the above equations become

$$\Delta\alpha_{DA} = \pm 0^s.0213 \rho \cos \varphi' / \cos \delta \quad (5\text{-}46)$$

$$\Delta\delta_{DA} = 0''.0 \quad (5\text{-}47)$$

where the positive sign corresponds to upper transits and the negative sign corresponds to lower transits. In practice, the increment as calculated from Eq. 5-46 is usually applied as a correction to the observed time of transit rather than as a correction to the right ascension of the star.

Table 5.2 gives values of the correction and the rule governing its application.

5.5.2 To determine the effect of the diurnal aberration on the azimuth and altitude of a body, consider Fig. 5.13, which is the same as Fig. 5.12 except that the aberration term $\kappa_d \sin\theta$ is shown resolved into the components $\Delta Z \cos a$ and $-\Delta a$. Again, since the aberration is small, the figure BRB' is so small that it may be considered as a plane triangle, so that we may say

$$\Delta Z \cos a = \kappa_d \sin\theta \sin\eta$$

$$\Delta Z = \kappa_d \sec a \sin\theta \sin\eta \quad (5\text{-}48)$$

$$-\Delta a = \kappa_d \sin\theta \cos\eta \quad (5\text{-}49)$$

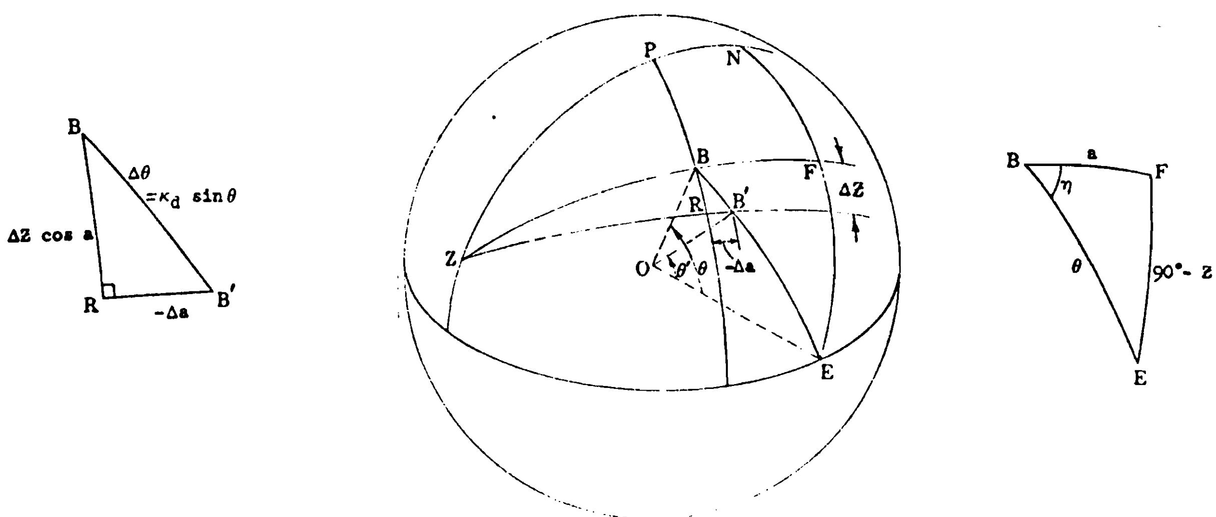


Fig. 5.13 The effect of diurnal aberration on azimuth and altitude.

In the spherical triangle BFE, using the law of sines, we obtain

$$\frac{\sin\theta}{\sin 90^\circ} = \frac{\sin(90^\circ - Z)}{\sin\eta}$$

$$\sin\theta\,\sin\eta = \cos Z \tag{5-50}$$

Applying the five-parts formula (Eq. 3-2) to the same figure, we get

$$\sin\theta\,\cos\eta = \cos(90^\circ - Z)\sin a - \sin(90^\circ - Z)\cos a\cos 90^\circ$$

$$\sin\theta\,\cos\eta = \sin Z\sin a \tag{5-51}$$

Substituting Eqs. 5-50 and 5-51 into Eqs. 5-48 and 5-49 respectively, and using the relationship between azimuth A and azimuth angle Z as discussed in section 3.5, we get

$$\Delta A_{DA} = A' - A = 0''.320\,\rho\cos\varphi'\cos A/\cos a \tag{5-52}$$

$$\Delta a_{DA} = a' - a = -0''.320\,\rho\cos\varphi'\sin A\sin a \tag{5-53}$$

X_o

Table 5.2 gives the correction, due to diurnal aberration, to the time of transit of heavenly bodies.

Table 5.2 CORRECTION FOR DIURNAL ABERRATION *

Dec.	Lat. 0°	10°	20°	30°	35°	40°	45°	50°	52°	54°	56°	58°	60°
°						Unit 0ˢ·001							
0	21	21	20	18	17	16	15	14	13	13	12	11	11
5	21	21	20	19	18	16	15	14	13	13	12	11	11
10	22	21	20	19	18	17	15	14	13	13	12	11	11
15	22	22	21	19	18	17	16	14	14	13	12	12	11
20	23	22	21	20	19	17	16	15	14	13	13	12	11
25	24	23	22	20	19	18	17	15	14	14	13	12	12
30	25	24	23	21	20	19	17	16	15	14	14	13	12
35	26	26	24	23	21	20	18	17	16	15	15	14	13
40	28	27	26	24	23	21	20	18	17	16	16	15	14
45	30	30	28	26	25	23	21	19	19	18	17	16	15
50	33	33	31	29	27	25	23	21	20	20	19	18	17
52	35	34	33	30	28	27	25	22	21	20	19	18	17
54	36	36	34	31	30	28	26	23	22	21	20	19	18
56	38	38	36	33	31	29	27	24	23	22	21	20	19
58	40	40	38	35	33	31	28	26	25	24	23	21	20
60	43	42	40	37	35	33	30	27	26	25	24	23	21
62	45	45	43	39	37	35	32	29	28	27	25	24	23
64	49	48	46	42	40	37	34	31	30	29	27	26	24
66	52	52	49	45	43	40	37	34	32	31	29	28	26
68	57	56	54	49	47	44	40	37	35	33	32	30	28
70	62	61	59	54	51	48	44	40	38	37	35	33	31
71	66	65	62	57	54	50	46	42	40	39	37	35	33
72	69	68	65	60	57	53	49	44	43	41	39	37	35
73	73	72	69	63	60	56	52	47	45	43	41	39	36
74	77	76	73	67	63	59	55	50	48	45	43	41	39
75	82	81	77	71	68	63	58	53	51	48	46	44	41
76	88	87	83	76	72	68	62	57	54	52	49	47	44
77	95	93	89	82	78	73	67	61	58	56	53	50	47
78	103	101	96	89	84	79	73	66	63	60	57	54	51
79	112	110	105	97	92	86	79	72	69	66	63	59	56
° ′						Unit 0ˢ·01							
80 00	12	12	12	11	10	9	9	8	8	7	7	7	6
81 00	14	13	13	12	11	10	10	9	8	8	8	7	7
82 00	15	15	14	13	13	12	11	10	9	9	9	8	8
83 00	18	17	16	15	14	13	12	11	11	10	10	9	9
84 00	20	20	19	18	17	16	14	13	13	12	11	11	10
85 00	24	24	23	21	20	19	17	16	15	14	14	13	12
85 30	27	27	26	24	22	21	19	17	17	16	15	14	14
86 00	31	30	29	26	25	23	22	20	19	18	17	16	15
86 30	35	34	33	30	29	27	25	22	22	21	20	19	17
87 00	41	40	38	35	33	31	29	26	25	24	23	22	20
87 30	49	48	46	42	40	37	35	31	30	29	27	26	24
88 00	61	60	57	53	50	47	43	39	38	36	34	32	31
88 10	67	66	63	58	55	51	47	43	41	39	37	35	33
88 20	73	72	69	64	60	56	52	47	45	43	41	39	37
88 30	82	80	77	71	67	62	58	52	50	48	46	43	41
88 40	92	90	86	79	75	70	65	59	56	54	51	49	46
88 50	105	103	98	91	86	80	74	67	65	62	59	56	52
89 00	122	120	115	106	100	94	86	79	75	72	68	65	61

This correction is to be *subtracted* from the observed time of transit for transits **above pole**, and *added* to the time of transit for transits below pole.

*From the "Explanatory Supplement to the Ephemeris", 1961

Station error

5.6 Deflection of the Vertical. As was discussed in section 2.3.4, the astronomic vertical and horizon do not, in general, coincide with the geodetic vertical and horizon, since the local gravity vertical does not, in general, point to the center of the earth; in fact, it may be that the gravity vertical does not even lie in the plane of the observer's geodetic meridian. Since measurements made with a gravity-sensing instrument, such as a transit or a theodolite, are necessarily referred to the astronomic (gravity) vertical and horizon, it is mandatory to be able to transform mathematically between the astronomic and the geodetic systems of coordinates.

The positive directions of the deflection coefficients ξ and η were defined in section 2.3.4 in such a way that the horizontal plate of the gravity-sensing instrument will be tilted down with respect to the geodetic horizon at the north and east points (based on taking east longitudes as positive). This is the situation shown in Fig. 5.6.1, with Z_g the geodetic zenith, Z_a the astronomic zenith, N_g and E_g the geodetic north and east points, respectively, and C the center of the earth.

The maximum tilt of the horizontal plate, i.e., the angle of inclination between the two horizons (and verticals), is denoted by τ ; the maximum downward tilt of the astronomic horizon (i.e., the instrument's horizontal plate) with respect to the geodetic horizon occurs at an azimuth υ . It may be shown that, for small τ ,

$$\tau = \sqrt{\xi^2 + \eta^2} \quad \text{and} \quad \upsilon = \tan^{-1}\left(\frac{\eta}{\xi}\right)$$

where $\xi = \varphi_a - \varphi_g$, and $\eta = (\lambda_a - \lambda_g)\cos\varphi_a$, from sec. 2.3.4.

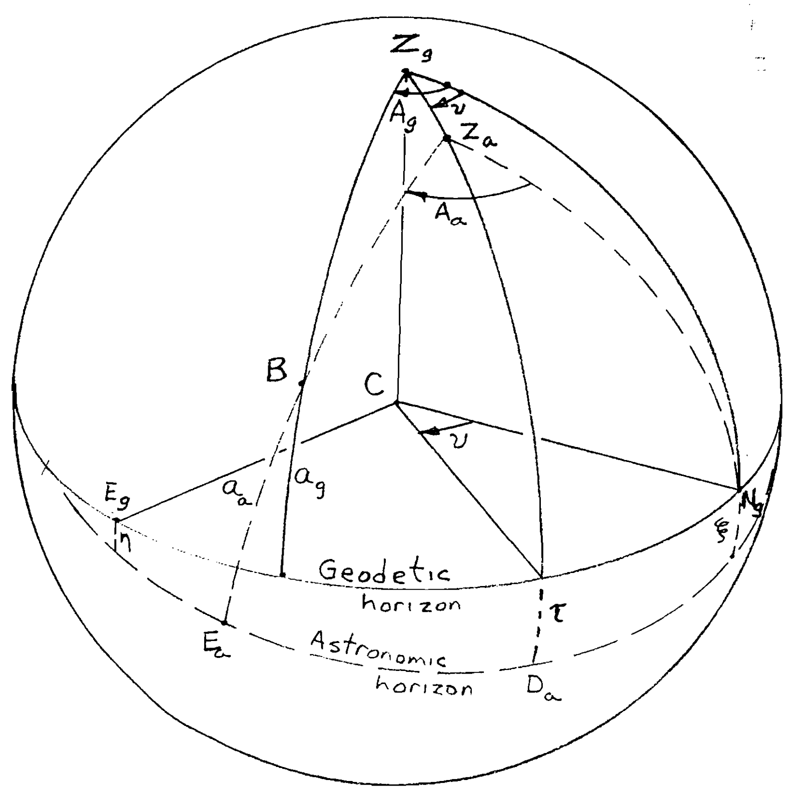


Fig. 5.6.1
Relations among astronomic and geodetic coordinates.

The spherical triangle $Z_g Z_a B$ is shown in Fig. 5.6.2. The interior angles at Z_g and Z_a are, by inspection of Fig. 5.6.1, equal to $A_g - \upsilon$ and $180° - (A_a - \upsilon)$, respectively. In the derivations which follow, it is assumed that ξ and η, and thus τ and υ, are known.

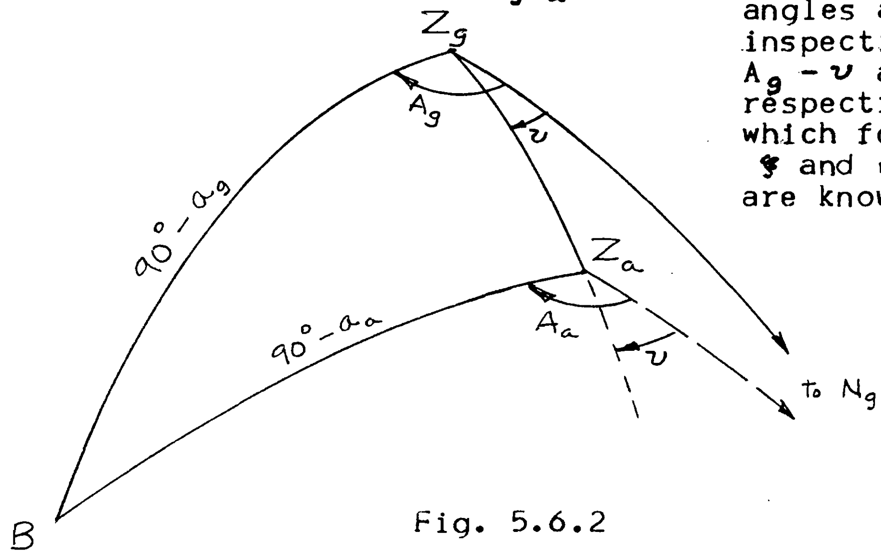


Fig. 5.6.2

Case 1 - Prediction of position.

The four-consecutive-parts formula, eq. 3-5, gives

$$\tan(180° - (A_a - \upsilon)) = \frac{\sin(A_g - \upsilon)}{\dfrac{\sin\tau}{\tan(90° - A_g)} - \cos\tau\,\cos(A_g - \upsilon)}$$

$$\tan(A_a - \upsilon) = \frac{\sin(A_g - \upsilon)}{-\sin\tau\,\tan A_g + \cos\tau\,\cos(A_g - \upsilon)} \qquad [5\text{-dv-1}]$$

$$A_a = (A_a - \upsilon) + \upsilon \qquad [5\text{-dv-2}]$$

The law of cosines gives

$$\cos(90° - a_a) = \cos(90° - a_g)\cos\tau + \sin(90° - a_g)\sin\tau\,\cos(A_g - \upsilon)$$

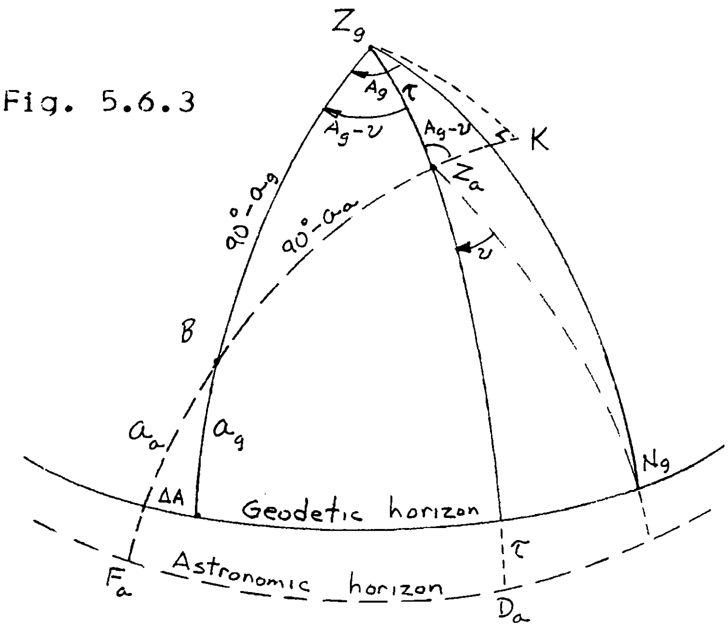
$$\sin a_a = \sin a_g \cos\tau + \cos a_g\,\sin\tau\,\cos(A_g - \upsilon) \qquad [5\text{-dv-2}]$$

While the preceding development is rigorous, the approximate method discussed below may be preferred. It uses the fact that τ is always small, the greatest value known being less than 40 arc-seconds.

Referring to Fig. 5.6.3, which is a skeletonized version of Fig. 5.6.1 with a few additional lines and labels to aid in the discussion to follow, the arc $Z_g K$ is a great circle which cuts the astronomic vertical BZ_a (extended) at K.

Since τ is small, the spherical triangle $Z_g Z_a K$ may be regarded as a plane triangle, and the interior angle at Z_a may be taken as equal to $A_g - \nu$, to sufficient accuracy. Then

Fig. 5.6.3



$$Z_g K = \tau \sin(A_g - \nu) \quad \text{and}$$

$$Z_a K = \tau \cos(A_g - \nu) \quad .$$

Using the rule that the arc distance between intersecting great circles is, for small angles of intersection, proportional to the sine of the distance from the point of intersection to the point of interest,

$$\frac{\Delta A}{\sin a_g} = \frac{Z_g K}{\sin(90^\circ - a_g)} = \frac{\tau \sin(A_g - \nu)}{\cos a_g}$$

$$\Delta A = A_a - A_g = \tau \tan a_g \sin(A_g - \nu) \qquad (5\text{-}54a)$$

$$A_a = A_g + \tau \tan a_g \sin(A_g - \nu) \qquad (5\text{-}54b)$$

Again using the rule stated above for intersecting circles, and since the distance $D_a F_a$ is very nearly equal to $A_g - \nu$,

$$\Delta a = a_a - a_g = \tau \sin(90^\circ - (A_g - \nu))$$

$$\Delta a = a_a - a_g = \tau \cos(A_g - \nu) \qquad (5\text{-}55a)$$

$$a_a = a_g + \tau \cos(A_g - \nu) \qquad (5\text{-}55b)$$

Case 2 - Reduction of observation.

Proceeding in similar fashion to Case 1, the following rigorous formulas may be obtained:

$$\tan(A_g - \upsilon) = \frac{\sin(A_a - \upsilon)}{\sin\tau \tan a_a + \cos\tau \cos(A_a - \upsilon)} \qquad [5\text{-dv-}7]$$

$$A_g = (A_g - \upsilon) + \upsilon \qquad [5\text{-dv-}8]$$

$$\sin a_g = \sin a_a \cos\tau - \cos a_a \sin\tau \cos(A_a - \upsilon) \qquad [5\text{-dv-}9]$$

The following approximate equations may also be obtained:

$$\Delta A_g = A_g - A_a = -\tau \tan a_a \sin(A_a - \upsilon) \qquad [5\text{-dv-}10]$$

$$\Delta a_g = a_g - a_a = -\tau \cos(A_a - \upsilon) \qquad [5\text{-dv-}11]$$

19.5989

Exercises

The observations referred to in the following problems are assumed as to be made, or having been made, with a perfectly adjusted theodolite. In each case, find the azimuth and altitude as indicated, including the effect of all appropriate corrections. Carry out the calculations to a precision of 0.″1 , but round off the final answer in each case to the nearest 1″ .

	5-1	5-2	5-3	5-4	5-5	5-6
Body	Sun	Jupiter	Moon	Altair	Sun	Moon
UTC day	Jun 17	Sep 10	Nov 3	Aug 24	Feb 19	May 9
UTC tod	14:40	03:35	21:30	23:15	05:15	09:44
Barom.	761mm	1000mb	625mm	692mm	995mb	615mm
Air tmp.	34 C	17 C	46 F	32 C	22 F	23 C
Azimuth	157° 19′46″	243° 17′56″	76° 34′ 22″	312° 59′08″	169° 44′ 12″	198° 26′ 33″
of limb	left	right	right	---	right	left
Alt.	22° 42′ 30″	28° 40′ 12″	14° 55′ 06″	37° 14′ 19″	73° 20′ 22″	21° 52′ 13″
of limb	lower	upper	lower	---	upper	lower
Obs lat	30° 12′ 15″	12° 27′ 37″	−41° 32′ 24″	−8° 09′ 10″	20° 30′ 40″	51° 06′ 48″
Hite	200m	847m	1660m	1050m	400m	1820m
Undul	+18m	−12m	+10m	−6m	−4m	+9m
DV lon	+9″	−15″	−6″	+8″	−5″	+13″
DV lat	−12″	+10″	−14″	−4″	+7″	+8″

5-7. Find the geodetic latitude of an observer who measures the meridian altitudes at upper and lower transits of a north circumpolar star as 55° 42′ 12″ N and 21° 11′ 36″ N, respectively. Assume other data as follows: barometer reads 750 mm Hg; thermometer reads 15 degrees C; height of observer above mean sea level is 1277 meters; no geoidal undulation or deflection of the vertical data is known.

5-8. Find the geodetic azimuth of the center of the sun at the instant on June 19 when the observed altitude of the lower limb is 30° 31′ 36″ and the observed azimuth of the left limb is 82° 44′ 30″. The UTC of the observation is $6^h 45^m$; the barometer is 782 mm Hg, and the air temperature is 13 C.

5-9. Find the geodetic azimuth and altitude of the center of the moon at the epoch May $11^d 5^h 0^m$ UTC, if the observed altitude of the upper limb was 25° 51′ 10″ and the observed azimuth of the right limb was 110° 46′ 08″. The barometer was read as 746 mm Hg and the air temperature was 18 degrees C. The observer was located at 2450 meters above msl; the undulation of the geoid at that point is known to be −15 meters; the deflection of the vertical is +10″ in longitude and −8″ in latitude.

5-10. From a place in latitude 20°N, a star having a declination of 45°N is observed at an hour angle of −30° ; find the corrections to both right ascension and declination due to the effect of diurnal aberration.

5-11. Same as 5-10, except the star is at transit.

5-12. From a place in latitude 30°N, a certain star is observed at an azimuth of 60° and an altitude of 45° ; find the diurnal aberration corrections to both azimuth and altitude.

5-13. Same as 5-12, except the star is at transit.

5-14. Find the obcentric azimuth and altitude of Venus for the following data:

observing site is the 42-inch Clark reflector at Lowell Observatory, Flagstaff, Arizona
(see the Appendix, Table B-5 for location data)
epoch is May 23 at 12:30 zone time −7
expected barometric pressure is 610 mm Hg
expected air temperature is 70 degrees Fahrenheit
deflection of the vertical in latitude is −3"
" " " " longitude is −7"
undulation of the geoid may be neglected

CHAPTER 6

STAR PLACES

6.1 Changes in the Coordinates of Stars. In the preceding chapters, it has been assumed that the coordinates of stars in the independent equatorial system (right ascension and declination) are constant with respect to time, and that in the dependent equatorial system (local hour angle and declination) and the horizon system (azimuth and altitude) the coordinates change continuously due only to the diurnal rotation of the earth. Careful observations carried out over long periods of time have shown, however, that all celestial coordinates suffer additional small changes because of the following factors:

(a) the motion of the coordinate fundamental circles with respect to the stars (precession and nutation--see section 4.2)

(b) the apparent displacements of the directions of the stars due to physical causes (refraction, parallax, and aberration--see Chap. 5); and

(c) the motion of the stars relative to each other in space (proper motion, defined as the continuous, slow change in angular position of a star as measured by reference to extremely remote background stars, which are considered as fixed. A few stars having exceptionally large proper motions are listed in Table 6.1; the proper motions of most stars are very much less than 1 second of arc per year.)

Table 6.1 Proper Motion of Stars

Star	Proper motion, arc-sec. per year
Barnard's Star	10.37 (largest known)
Kapteyn's Star	8.8
Groombridge 1830	7.1
Lacaille 9352	6.9
Cordoba 32416	6.1
61 Cygni A	5.21
α Centauri A	3.67
van Maanen's Star	3.01
Arcturus	2.283
Sirius A	1.32

Two of the above effects (refraction and diurnal aberration) are treated as corrections to observations, as discussed in Chapter 5; the remainder (precession, proper motion, nutation, annual aberration, and annual parallax) are considered as changes in the coordinates themselves. The purpose of the present discussion is to explain the system of star positions, or places, currently in use, and to show how to obtain the correct position of a specific star for a specific observer and epoch.

6.2 Star Places. In order to distinguish between the several kinds of coordinates of stars, the adjectives observed, apparent, and mean are used; they are defined as follows:

The observed place of a star is its position as viewed from an observer (who is usually, but not necessarily, located on or near the surface of the earth) by means of an instrument which is free from error.

The apparent place of a star is its geocentric position (i.e., as viewed by an imaginary observer at the center of the earth) referred to the instantaneous (true) equinox and equator at the epoch of the observation. It differs from the observed place by the effects of refraction and diurnal aberration. (Geocentric parallax is negligible for stars due to their very great distances from the earth.) Apparent places of 1,535 stars are given in the publication "Apparent Places of Fundamental Stars", an annual volume published by the Astronomisches Rechen-Institut in Heidelberg, West Germany.

The mean place of a star is its heliocentric position (i.e., as it would be viewed by an imaginary observer at the center of the sun) referred to a specific mean equinox and mean equator. When the reference equinox and equator are those at the beginning of a so-called Besselian solar year, defined as that instant near the beginning of a Gregorian calendar year when the right ascension of the mean sun is 18h 40m exactly, the usual practice is to describe the star position as being "the mean place for the epoch B1950.0", or whatever year is appropriate. The Astronomical Almanac gives the mean places of stars at the middle of the current Julian year, referred to the mean equinox and mean equator of the epoch 2000 January 1d 12h UTC, designated as J2000.0 and also as 2000 January 1.5 .

The apparent place differs from the mean place by the effects of precession and proper motion over the time interval from the mean place epoch to the epoch of observation, and also by the effects of nutation, annual aberration, and annual parallax as described briefly in sections 4.2, 5.3 and 5.5 and more fully in sections 6.6, 6.7, and 6.8 , following. Mean places of 1,482 stars are given in the Almanac, listed in order of increasing right ascension. In the name of each listed star, the 3-letter abbreviations for constellation names as recommended by the International Astronomical Union are used. A list of these constellation names and abbreviations is given in Appendix C of this text.

6.3 Reduction from Mean Place to Apparent Place. In the types of problems usually encountered in Engineering Astronomy, it is often necessary to have the apparent (geocentric) place of a star; this is most readily taken from a publication such as "Apparent Places of Fundamental Stars", mentioned above. It may be, however, that such a volume is not available so that it becomes necessary to convert an Almanac mean place at mid-year to the desired apparent place at the epoch of observation. This conversion process is called the reduction from mean to apparent place, and includes corrections for each of the five effects (precession, proper motion, nutation, annual aberration, and annual parallax) mentioned above. Each of these effects is discussed separately in the sections which follow.

6.4 Precession. The equator and the ecliptic are continuously in motion, as was pointed out in section 4.2 . The motion of the equator, and therefore of the celestial poles, is due to the gravitational attraction of the sun and the moon on the earth's equatorial bulge. This motion consists of two components: (1) the luni-solar precession, which is the smooth, very-long-period (about 26,000 years) motion of the mean celestial pole around the pole of the ecliptic, and (2) the nutation, which is a relatively short-period motion which carries the actual (i.e., true, instantaneous) celestial pole around the mean celestial pole in a somewhat irregular curve, of amplitude about 9" and main period 18.6 years.

The motion of the ecliptic, that is, of the mean plane of the earth's orbit, is due to the gravitational attraction of the planets on the earth as a whole, and consists of a slow rotation of the ecliptic about a slowly-moving diameter. This motion is called the planetary precession, and results in a westward movement of the equinox of about 12" a century and a decrease in the obliquity of about 47" a century.

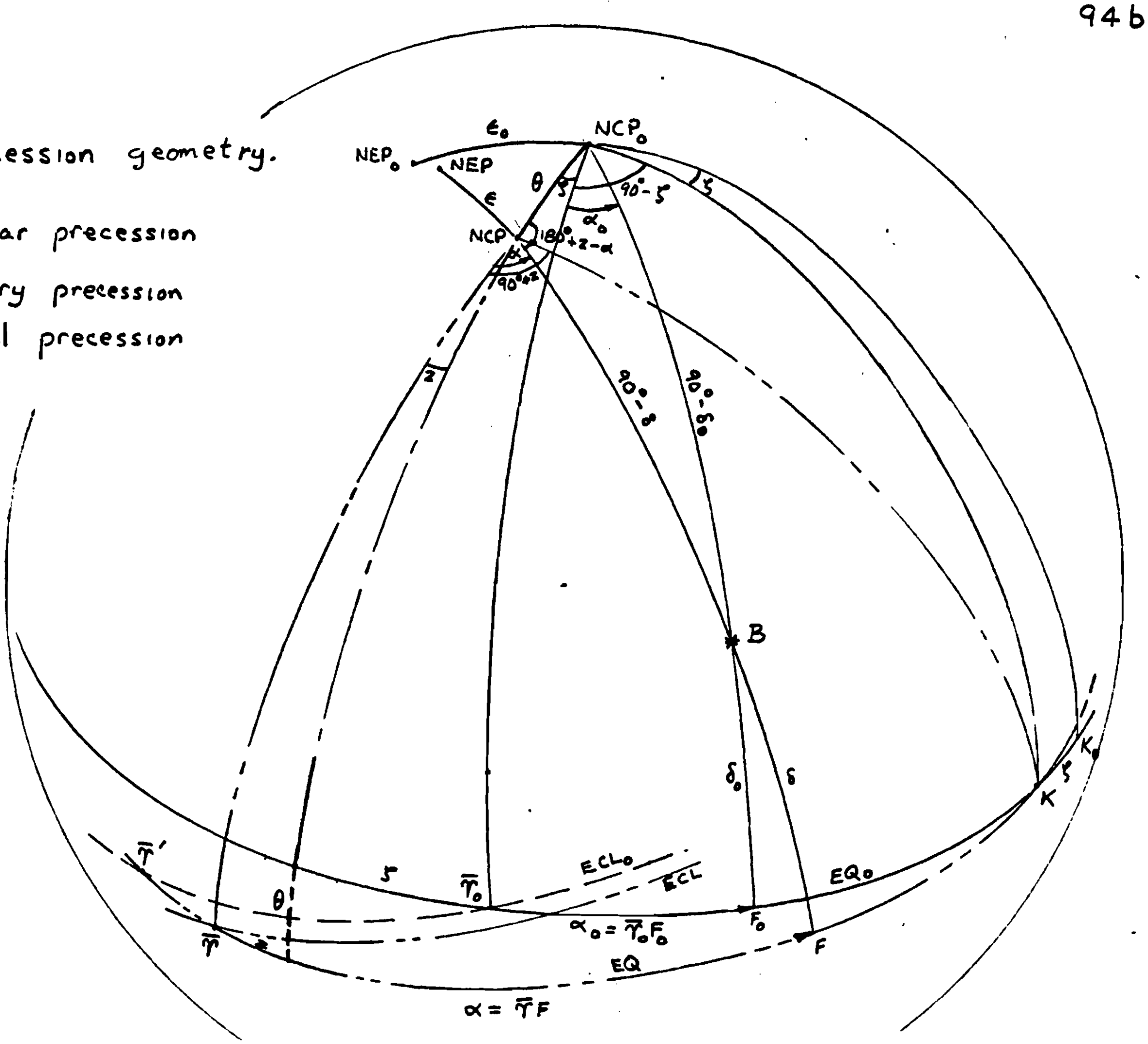
The combination of the luni-solar and the planetary precessions is called general precession, and is the subject of the remaining discussion in the present section. The effect of the nutation is discussed separately in section 6.6 .

6.4.1 General precession and the precessional angles. The effect of precession on the coordinates of a fixed point is illustrated in Fig. 6.1, in which the position of a body at B is referred at an initial epoch t_0 to a coordinate system defined by the mean equator EQ_0 and the mean ecliptic ECL_0; the intersection of the two fundamental great circles is the mean equinox Υ_0 . The poles NCP_0 and NEP_0 of the mean equator and ecliptic, respectively, are also shown. At a later epoch t , the positions of the (mean) equator, ecliptic, equinox, and poles are shown as EQ, ECL, $\overline{\Upsilon}$, NCP, and NEP, respectively.

Although at any instant NCP moves, due to the luni-solar precession, in a direction perpendicular to the arc NEP-NCP, i.e., towards the equinox, the arc NCP_0-NCP is not perpendicular to either NEP_0-NCP or to NEP-NCP; because of the planetary precession, NEP is itself in motion along a curve which is always convex to NEP-NCP. This complex motion is specified by the three angles ζ , z , and θ , also shown in the figure. It should be noted that θ is not the actual path taken by the moving pole NCP, but is merely the great circle arc connecting two discrete points of that path, viz., NCP_0 and NCP.

Fig. 6.1 Precession geometry.

$\bar{\Upsilon}_0\bar{\Upsilon}'$ = luni-solar precession
$\bar{\Upsilon}'\bar{\Upsilon}$ = planetary precession
$\bar{\Upsilon}_0\bar{\Upsilon}$ = general precession



For the reduction of mean places based on the FK4 system, and with time measured in tropical centuries of 36524.22 years from the fundamental epoch B1950.0 = 1950 January 0.9 = JD 243 3282.423, the following formulas for the precessional angles are valid:

$$\zeta = 2304''.948\ T + 0''.302\ T^2 + 0''.0179\ T^3 \qquad (6\text{-}1)$$
$$z = 2304''.948\ T + 1''.093\ T^2 + 0''.0192\ T^3 \qquad (6\text{-}2)$$
$$\theta = 2004''.255\ T - 0''.426\ T^2 - 0''.0416\ T^3 \qquad (6\text{-}3)$$

where T = (JD-243 3282.423)/36524.22

For the reduction of mean places based on the FK5 system, and with time measured in Julian centuries of 36525 years from the fundamental epoch J2000.0 = 2000 January 1.5 = JD 245 1545.0, the following formulas for the precessional angles are valid:

$$\zeta_A = 0^\circ.640\ 6161\ T + 0^\circ.000\ 0839\ T^2 + 0^\circ.000\ 0050\ T^3 \qquad (6\text{-}4)$$
$$z_A = 0^\circ.640\ 6161\ T + 0^\circ.000\ 3041\ T^2 + 0^\circ.000\ 0051\ T^3 \qquad (6\text{-}5)$$
$$\theta_A = 0^\circ.556\ 7530\ T - 0^\circ.000\ 1185\ T^2 - 0^\circ.000\ 0116\ T^3 \qquad (6\text{-}6)$$

where T = (JD - 245 1545.0)/36525

The subscripts A in eqs. 6-4, 6-5, and 6-6 are used to distinguish these precessional values from the ones based on the FK4 system.

6.4.2 <u>Reduction of mean places-- rigorous formulas.</u>
Figure 6.3 is a portion of Figure 6.1, enlarged and simplified for clarity.

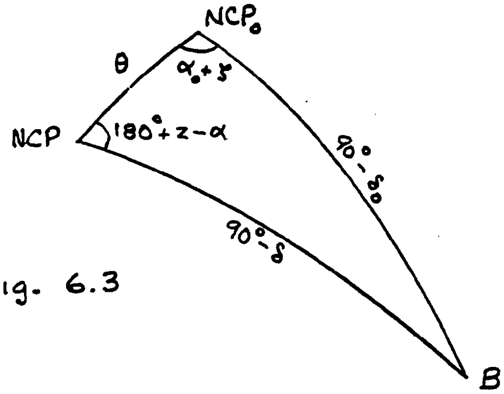


Fig. 6.3

In this triangle, the vertex angle at NCP may be found from the four-consecutive parts formula,

$$\tan(180^\circ + z - \alpha) = \frac{\sin(\alpha_0 + \zeta)}{\dfrac{\sin\theta}{\tan(90^\circ - \delta_0)} - \cos\theta\cos(\alpha_0 + \zeta)}$$

$$\tan(\alpha - z) = \frac{\sin(\alpha_0 + \zeta)}{-\sin\theta\tan\delta_0 + \cos\theta\cos(\alpha_0 + \zeta)} \qquad (6\text{-}7)$$

$$\alpha = (\alpha - z) + z \qquad (6\text{-}8)$$

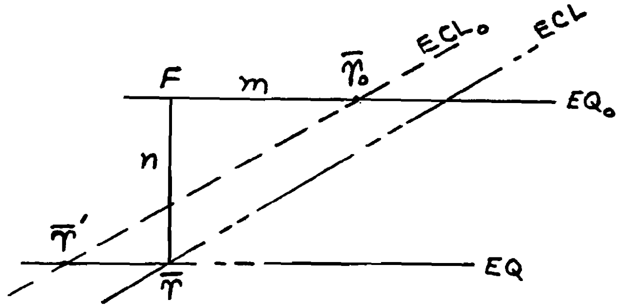
In the same spherical triangle, from the law of cosines,

$$\cos(90^\circ - \delta) = \cos\theta\cos(90^\circ - \delta_0) + \sin\theta\sin(90^\circ - \delta_0)\cos(\alpha_0 + \zeta)$$

$$\sin\delta = \cos\theta\sin\delta_0 + \sin\theta\cos\delta_0\cos(\alpha_0 + \zeta) \qquad (6\text{-}9)$$

6.4.3 <u>Annual motions and approximate formulae.</u> The precessional motions during a time interval of the order of 1 year or less are so small that the portion of Fig. 6.1 in the vicinity of the equinoxes may be regarded as a plane, as shown in Fig. 6.4 .

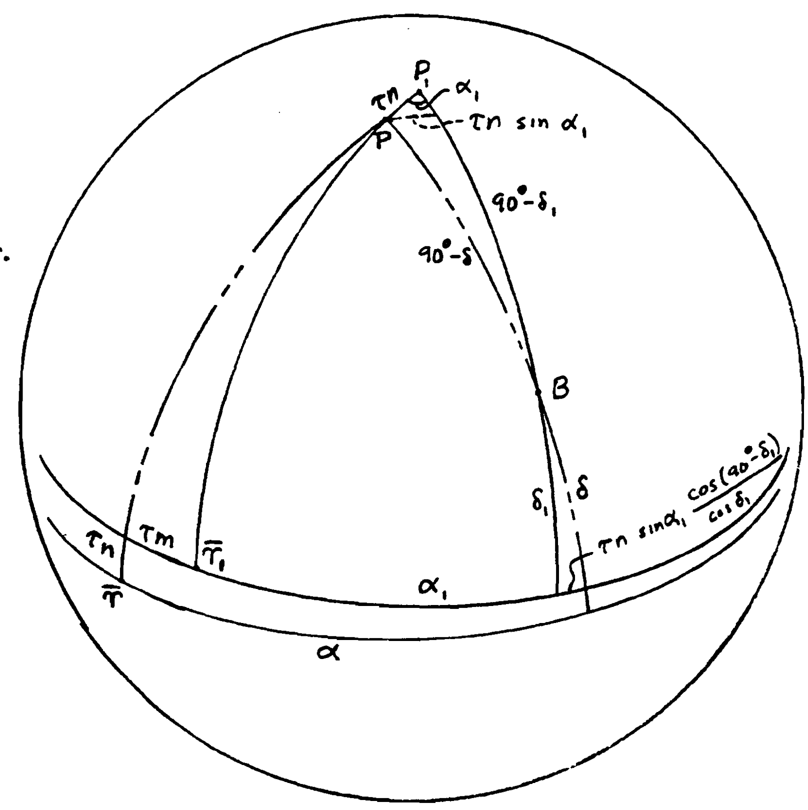
Fig. 6.4
Annual precession.



From a comparison of the figures, it is seen that the distance $m = F\overline{\Upsilon}_0$ in Fig. 6.4 is equal to the annual change in the sum $\zeta + z$ of Fig. 6.1; the quantity m is called the annual precession in right ascension. Similarly, it is seen that the distance $n = F\overline{\Upsilon}$ in Fig. 6.4 is equal to the annual change in the quantity θ in Fig. 6.1; the quantity n is called the annual precession in declination.

The amounts of precession in right ascension and declination for intervals of time less than one year are then obtained by multiplying these annual rates by τ, the decimal fraction of a year representing the interval. Figure 6.5 illustrates the situation.

Fig. 6.5 Annual precession.
Subscript 1 refers to the starting epoch within a given year.



By inspection of Fig. 6.5, keeping in mind that the distances τm and τn are small,

$$\alpha = \alpha_1 + \tau m + \tau n \sin \alpha_1 \frac{\sin \delta_1}{\cos \delta_1}$$

$$= \alpha_1 + \tau m + \tau n \sin \alpha_1 \tan \delta_1$$

$$\alpha = \alpha_1 + \tau (m + n \sin \alpha_1 \tan \delta_1) \qquad (6-10)$$

$$\delta = \delta_1 + \tau n \cos \alpha_1 \qquad (6-11)$$

Until 1984, the mean places of stars tabulated in the annual volumes of the Astronomical Almanac were given for the beginning of the current Besselian solar year, and were referred to the FK4 system. Beginning with the 1984 volume, the mean places are those for the middle of the current Julian year, this epoch being denoted as t_1 = 19YY.5 (where YY stands for the last two digits of the year), and are referred to the FK5 system. Reduction of these values to the mean equator and equinox at any other epoch t within the year may be done by means of eqs. 6-10 and 6-11. Values of the annual precessional constants m and n, required for solution of these equations, are obtained in the manner explained in the material which follows.

Referring to the precessional angles as defined in section 6.4.1, let

$$M = \zeta_A + z_A = 1^\circ.281\ 2323\ T + 0^\circ.000\ 3879\ T^2 + 0^\circ.000\ 0101\ T^3 \qquad (6-12)$$
$$N = \theta_A = 0^\circ.556\ 7530\ T - 0^\circ.000\ 1185\ T^2 - 0^\circ.000\ 0116\ T^3 \qquad (6-13)$$

Differentiating,

$$\dot{M} = dM/dT = 1^\circ.281\ 2323 + 0^\circ.000\ 7758\ T + 0^\circ.000\ 0303\ T^2 \qquad (6-14)$$
$$\dot{N} = dN/dT = 0^\circ.556\ 7530 - 0^\circ.000\ 2370\ T - 0^\circ.000\ 0348\ T^2 \qquad (6-15)$$

The preceding are the centennial values; for annual values, divide by 100 to obtain

$$m = \dot{M}/100 = 0^\circ.012\ 812323 + 0^\circ.000\ 007758\ T + 0^\circ.000\ 000303\ T^2 \qquad (6-16)$$
$$n = \dot{N}/100 = 0^\circ.005\ 567530 - 0^\circ.000\ 002370\ T - 0^\circ.000\ 000348\ T^2 \qquad (6-17)$$

6.5 Proper Motion. The term "proper motion" refers to the motion thru space of a star, as viewed against the essentially fixed background stars which are so extremely far away that they have no discernible motion of their own, and are referred to as "fixed stars". The spatial motion of a star may be considered as having two components, one at right angles to the star-earth line (or the star-sun line, which is the same, for practical purposes), and one in the star-earth line. It is the first of these with which we are concerned, and it is this component which is called the "proper motion" of the star. The other component is called the "radial motion", and is of no interest to us.

Proper motions of stars are small, the largest one known being only of the order of 10" per year, as is shown in Table 6.1 . This table is not exhaustive, and shows only a few of the larger proper motions. For proper motions of a specific star, consult one of the various star catalogs such as the FK4 catalog mentioned earlier.

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6.6 Nutation. Nutation is essentially that part of the precessional motion of the pole of the earth's equator which depends on the periodic motions of the sun and the moon in their orbits around the earth. The progressive long-period motion of the mean pole has been considered as the luni-solar precession in section 6.4; the nutation is the somewhat irregular elliptical motion of the true pole about the mean pole in a period of about 19 years with an amplitude of about 9" . The principal term depends on the longitude of the moon's orbit and has a period of 6798 days (18.6 years); the amplitude of this term, 9".2025, is known as the constant of nutation. In the complicated theory of the gravitational action of the sun and moon on the rotating, non-spherical, non-rigid earth, other terms arise which depend on the mean longitudes and mean anomalies of the sun and moon and on their combinations with the longitude of the node of the moon's orbit. The resulting shift of the mean to the true pole can be resolved into corrections to the mean longitude ($\Delta\psi$, the nutation in longitude) and to the mean obliquity ($\Delta\epsilon$, the nutation in obliquity). Expressions for these corrections, in the form of series, constitute the formal specification of the nutation.

When working with the FK4 system and the fundamental epoch of B1950.0, the series used is that developed by E. Woolard and others, published as "Astronomical Papers Prepared for the use of the American Ephemeris and Nautical Almanac", Volume XV, Part I, 1953; the series is also given in the "Explanatory Supplement to the Ephemeris, Section 2C. When working with the FK5 system and the fundamental epoch of J2000.0, the series used is that adopted by the International Astronomical Union and denoted as the "1980 IAU Theory of Nutation"; this series is published in the 1984 volume of the "Astronomical Almanac" and also in the United States Naval Observatory Circular No. 163, "The IAU Resolutions on Astronomical Constants, Time Scales, and the Fundamental Reference Frame", 1981. It is also presented as Table 6.2 of this text.

The terms divide naturally into those not depending on the moon's longitude, which can be interpolated to high accuracy at intervals of 10 days, and those which do depend on the moon's longitude, with periods of less than about 60 days, which cannot be so interpolated. Nutation is therefore conventionally divided into long-period terms and short-period terms; the latter, consisting of terms with periods of less than 35 days, are sometimes summed separately as $d\psi$ and $d\epsilon$, called the short-period terms of nutation in longitude and obliquity, respectively. In certain special applications, such as the tabulation of star positions at intervals of 10 days (e.g., in the volume "Apparent Places of Fundamental Stars"), only the long-period terms of nutation are included in the tabular places; data is provided, however, for the individual application of corrections for the effect of the much smaller short-period terms, after interpolation.

The IAU-80 nutation series contains 106 terms in $\Delta\psi$, of which 30 are long-period and 76 are short-period; there are 64 terms in $\Delta\epsilon$, of which 18 are long-period and 46 are short-period. In Table 6.2, the terms of the series are listed in descending order of size of the coefficient of the sine terms, within each period group.

Table 6.2 1980 IAU Theory of Nutation

Series for nutation in longitude $\Delta\psi$ and obliquity $\Delta\epsilon$, referred to the mean equator and equinox of date, with T measured in Julian centuries from epoch J2000.0.

r·T   −103808.7818   −.00002

Fundamental arguments

$l = 485866''.733 + (1325^r + 715922''.633)T + 31''.310T^2 + 0''.064T^3$

$l' = 1287099''.804 + (99^r + 1292581''.224)T - 0''.577T^2 - 0''.012T^3$

$F = 335778''.877 + (1342^r + 295263''.137)T - 13''.257T^2 + 0''.011T^3$

$D = 1072261''.307 + (1236^r + 1105601''.328)T - 6''.891T^2 + 0''.019T^3$

$\Omega = 450160''.280 - (5^r + 482890''.539)T + 7''.455T^2 + 0''.008T^3$

where $1^r = 360° = 1296000''$

- l is the mean anomaly of the Moon.
- l' is the mean anomaly of the Sun (Earth).
- Ω is the longitude of the ascending node of the Moon's mean orbit on the ecliptic, measured from the mean equinox of date.
- D is the mean elongation of the Moon from the Sun.
- F is the difference L—Ω, where L is the mean longitude of the Moon.

	ARGUMENT l	l'	F	D	Ω	PERIOD (DAYS)	−Δψ LONGITUDE (0″.0001)		Δε OBLIQUITY (0″.0001)	
1	0	0	0	0	1	6798.4	−171996	−174.2T	92025	8.9T
2	0	0	0	0	2	3399.2	2062	0.2T	−895	0.5T
3	−2	0	2	0	1	1305.5	46	0.0T	−24	0.0T
4	2	0	−2	0	0	1095.2	11	0.0T	0	0.0T
5	−2	0	2	0	2	1615.7	−3	0.0T	1	0.0T
6	1	−1	0	−1	0	3232.9	−3	0.0T	0	0.0T
7	0	−2	2	−2	1	6786.3	−2	0.0T	1	0.0T
8	2	0	−2	0	1	943.2	1	0.0T	0	0.0T
9	0	0	2	−2	2	182.6	−13187	−1.6T	5736	−3.1T
10	0	1	0	0	0	365.3	1426	−3.4T	54	−0.1T
11	0	1	2	−2	2	121.7	−517	1.2T	224	−0.6T
12	0	−1	2	−2	2	365.2	217	−0.5T	−95	0.3T
13	0	0	2	−2	1	177.8	129	0.1T	−70	0.0T
14	2	0	0	−2	0	205.9	48	0.0T	1	0.0T
15	0	0	2	−2	0	173.3	−22	0.0T	0	0.0T
16	0	2	0	0	0	182.6	17	−0.1T	0	0.0T
17	0	1	0	0	1	386.0	−15	0.0T	9	0.0T
18	0	2	2	−2	2	91.3	−16	0.1T	7	0.0T
19	0	−1	0	0	1	346.6	−12	0.0T	6	0.0T
20	−2	0	0	2	1	199.8	−6	0.0T	3	0.0T
21	0	−1	2	−2	1	346.6	−5	0.0T	3	0.0T
22	2	0	0	−2	1	212.3	4	0.0T	−2	0.0T
23	0	1	2	−2	1	119.6	4	0.0T	−2	0.0T
24	1	0	0	−1	0	411.8	−4	0.0T	0	0.0T
25	2	1	0	−2	0	131.7	1	0.0T	0	0.0T

(continued)

Table 6.2 (cont'd.)

	ARGUMENT					PERIOD	LONGITUDE		OBLIQUITY	
	l	*l'*	*F*	*D*	Ω	(DAYS)	(0.″0001)		(0.″0001)	
26	0	0	−2	2	1	169.0	1	0.0*T*	0	0.0*T*
27	0	1	−2	2	0	329.8	−1	0.0*T*	0	0.0*T*
28	0	1	0	0	2	409.2	1	0.0*T*	0	0.0*T*
29	−1	0	0	1	1	388.3	1	0.0*T*	0	0.0*T*
30	0	1	2	−2	0	117.5	−1	0.0*T*	0	0.0*T*
31	0	0	2	0	2	13.7	−2274	−0.2*T*	977	−0.5*T*
32	1	0	0	0	0	27.6	712	0.1*T*	−7	0.0*T*
33	0	0	2	0	1	13.6	−386	−0.4*T*	200	0.0*T*
34	1	0	2	0	2	9.1	−301	0.0*T*	129	−0.1*T*
35	1	0	0	−2	0	31.8	−158	0.0*T*	−1	0.0*T*
36	−1	0	2	0	2	27.1	123	0.0*T*	−53	0.0*T*
37	0	0	0	2	0	14.8	63	0.0*T*	−2	0.0*T*
38	1	0	0	0	1	27.7	63	0.1*T*	−33	0.0*T*
39	−1	0	0	0	1	27.4	−58	−0.1*T*	32	0.0*T*
40	−1	0	2	2	2	9.6	−59	0.0*T*	26	0.0*T*
41	1	0	2	0	1	9.1	−51	0.0*T*	27	0.0*T*
42	0	0	2	2	2	7.1	−38	0.0*T*	16	0.0*T*
43	2	0	0	0	0	13.8	29	0.0*T*	−1	0.0*T*
44	1	0	2	−2	2	23.9	29	0.0*T*	−12	0.0*T*
45	2	0	2	0	2	6.9	−31	0.0*T*	13	0.0*T*
46	0	0	2	0	0	13.6	26	0.0*T*	−1	0.0*T*
47	−1	0	2	0	1	27.0	21	0.0*T*	−10	0.0*T*
48	−1	0	0	2	1	32.0	16	0.0*T*	−8	0.0*T*
49	1	0	0	−2	1	31.7	−13	0.0*T*	7	0.0*T*
50	−1	0	2	2	1	9.5	−10	0.0*T*	5	0.0*T*
51	1	1	0	−2	0	34.8	−7	0.0*T*	0	0.0*T*
52	0	1	2	0	2	13.2	7	0.0*T*	−3	0.0*T*
53	0	−1	2	0	2	14.2	−7	0.0*T*	3	0.0*T*
54	1	0	2	2	2	5.6	−8	0.0*T*	3	0.0*T*
55	1	0	0	2	0	9.6	6	0.0*T*	0	0.0*T*
56	2	0	2	−2	2	12.8	6	0.0*T*	−3	0.0*T*
57	0	0	0	2	1	14.8	−6	0.0*T*	3	0.0*T*
58	0	0	2	2	1	7.1	−7	0.0*T*	3	0.0*T*
59	1	0	2	−2	1	23.9	6	0.0*T*	−3	0.0*T*
60	0	0	0	−2	1	14.7	−5	0.0*T*	3	0.0*T*
61	1	−1	0	0	0	29.8	5	0.0*T*	0	0.0*T*
62	2	0	2	0	1	6.9	−5	0.0*T*	3	0.0*T*
63	0	1	0	−2	0	15.4	−4	0.0*T*	0	0.0*T*
64	1	0	−2	0	0	26.9	4	0.0*T*	0	0.0*T*
65	0	0	0	1	0	29.5	−4	0.0*T*	0	0.0*T*
66	1	1	0	0	0	25.6	−3	0.0*T*	0	0.0*T*
67	1	0	2	0	0	9.1	3	0.0*T*	0	0.0*T*
68	1	−1	2	0	2	9.4	−3	0.0*T*	1	0.0*T*
69	−1	−1	2	2	2	9.8	−3	0.0*T*	1	0.0*T*
70	−2	0	0	0	1	13.7	−2	0.0*T*	1	0.0*T*

(continued)

Table 6.2 (cont'd.)

	ARGUMENT					PERIOD	LONGITUDE		OBLIQUITY	
	l	l'	F	D	Ω	(DAYS)	(0."0001)		(0."0001)	
71	3	0	2	0	2	5.5	−3	0.0T	1	0.0T
72	0	−1	2	2	2	7.2	−3	0.0T	1	0.0T
73	1	1	2	0	2	8.9	2	0.0T	−1	0.0T
74	−1	0	2	−2	1	32.6	−2	0.0T	1	0.0T
75	2	0	0	0	1	13.8	2	0.0T	−1	0.0T
76	1	0	0	0	2	27.8	−2	0.0T	1	0.0T
77	3	0	0	0	0	9.2	2	0.0T	0	0.0T
78	0	0	2	1	2	9.3	2	0.0T	−1	0.0T
79	−1	0	0	0	2	27.3	1	0.0T	−1	0.0T
80	1	0	0	−4	0	10.1	−1	0.0T	0	0.0T
81	−2	0	2	2	2	14.6	1	0.0T	−1	0.0T
82	−1	0	2	4	2	5.8	−2	0.0T	1	0.0T
83	2	0	0	−4	0	15.9	−1	0.0T	0	0.0T
84	1	1	2	−2	2	22.5	1	0.0T	−1	0.0T
85	1	0	2	2	1	5.6	−1	0.0T	1	0.0T
86	−2	0	2	4	2	7.3	−1	0.0T	1	0.0T
87	−1	0	4	0	2	9.1	1	0.0T	0	0.0T
88	1	−1	0	−2	0	29.3	1	0.0T	0	0.0T
89	2	0	2	−2	1	12.8	1	0.0T	−1	0.0T
90	2	0	2	2	2	4.7	−1	0.0T	0	0.0T
91	1	0	0	2	1	9.6	−1	0.0T	0	0.0T
92	0	0	4	−2	2	12.7	1	0.0T	0	0.0T
93	3	0	2	−2	2	8.7	1	0.0T	0	0.0T
94	1	0	2	−2	0	23.8	−1	0.0T	0	0.0T
95	0	1	2	0	1	13.1	1	0.0T	0	0.0T
96	−1	−1	0	2	1	35.0	1	0.0T	0	0.0T
97	0	0	−2	0	1	13.6	−1	0.0T	0	0.0T
98	0	0	2	−1	2	25.4	−1	0.0T	0	0.0T
99	0	1	0	2	0	14.2	−1	0.0T	0	0.0T
100	1	0	−2	−2	0	9.5	−1	0.0T	0	0.0T
101	0	−1	2	0	1	14.2	−1	0.0T	0	0.0T
102	1	1	0	−2	1	34.7	−1	0.0T	0	0.0T
103	1	0	−2	2	0	32.8	−1	0.0T	0	0.0T
104	2	0	0	2	0	7.1	1	0.0T	0	0.0T
105	0	0	2	4	2	4.8	−1	0.0T	0	0.0T
106	0	1	0	1	0	27.3	1	0.0T	0	0.0T

<u>6.6.1 Evaluation of the series for the nutation.</u> To evaluate the series for a specific epoch, first find the time argument T in Julian centuries from the fundamental epoch, J2000.0 , then compute the values of the five fundamental arguments listed below:

$$\begin{aligned}
l &= 485866.''733 + (1325^r + 715922.''633)T + 31.''310\,T^2 + 0.''064\,T^3 && (6\text{-}22)\\
l' &= 1287099.''804 + (\;\;99^r + 1292581.''224)T - 0.''577\,T^2 - 0.''012\,T^3 && (6\text{-}23)\\
F &= 335778.''877 + (1342^r + 295263.''137)T - 13.''257\,T^2 + 0.''011\,T^3 && (6\text{-}24)\\
D &= 1072261.''307 + (1236^r + 1105601.''328)T - 6.''891\,T^2 + 0.''019\,T^3 && (6\text{-}25)\\
\Omega &= 450160.''280 - (\;\;\;5^r + 482890.''539)T + 7.''455\,T^2 + 0.''008\,T^3 && (6\text{-}26)
\end{aligned}$$

The series is then evaluated term by term, as indicated below.

The first few terms of the series for the nutation in longitude are:

$$\begin{aligned}
\Delta\psi = &\;(-17.''1996 - 0.''01742\,T)\sin(\Omega)\\
&+(.''2062 + .''00002\,T)\sin(2\Omega)\\
&+(.''0046)\sin(-2l + 2F + \Omega)\\
&+(.''0011)\sin(2l - 2F)\\
&\quad\text{etc.}
\end{aligned}$$

The first few terms of the series for the nutation in obliquity are:

$$\begin{aligned}
\Delta\epsilon = &\;(9.''2025 + 0.''00089\,T)\cos(1\Omega)\\
&+(-.''0895 + .''00005\,T)\cos(2\Omega)\\
&+(-.''0024)\cos(-2l + 2F + 1\Omega)\\
&+(.''0001)\cos(-2l + 2F + 2\Omega)\\
&\quad\text{etc.}
\end{aligned}$$

The nutation in longitude, to be added to longitudes referred to the mean equinox of date, is tabulated to $0.''001$ for 0^h Dynamical Time on each day of the year in the Astronomical Almanac, Section B, in the subsection entitled "Nutation, Obliquity, Day Numbers". The nutation in obliquity,to be added to the mean obliquity of date, is also tabulated there, to the same precision.

The equation of the equinoxes is the right ascension of the mean equinox referred to the true (apparent) equinox. It is equal to $\Delta\psi \cos\epsilon$ and represents the difference between the mean and true (apparent) right ascensions of a heavenly body, in the sense of apparent minus mean. It is thus also equal to the difference between apparent and mean sidereal times. The equation of the equinoxes (EOE) is tabulated to $0.^s0001$ in the Almanac, Section B, in the subsection entitled "Universal and Sidereal Times", and is included in the apparent sidereal times given on the same pages.

<u>6.6.2 Correction to coordinates for the effect of nutation.</u>
The simplest and most direct way to convert a position from the mean equinox and mean equator (mean place) to the true equinox and true equator (apparent place) for the effect of nutation is to add $\Delta\psi$ to the ecliptic longitude, since the ecliptic, and therefore the latitude, is unchanged by nutation. In making the conversion from the resultant ecliptic coordinates to the corresponding equatorial ones, by means of the usual formulas of spherical trigonometry, the true obliquity $\epsilon = \bar{\epsilon} + \Delta\epsilon$ should be used. If the mean place right ascension and declination are already known, direct corrections to these values for the effect of nutation may be made in the manner described in the paragraphs which follow.

6.6.1 Evaluation of the series for the nutation. To evaluate the series for a specific epoch, first find the time argument T in Julian centuries from the fundamental epoch, J2000.0 , then compute the values of the five fundamental arguments listed below:

$$l = 485866''.733 + (1325^{r} + 715922''.633)T + 31''.310\,T + 0''.064\,T \quad (6\text{-}22)$$

$$l' = 1287099''.804 + (\;\;99^{r} + 1292581''.224)T - 0''.577\,T - 0''.012\,T \quad (6\text{-}23)$$

$$F = 335778''.877 + (1342^{r} + 295263''.137)T - 13''.257\,T + 0''.011\,T \quad (6\text{-}24)$$

$$D = 1072261''.307 + (1236^{r} + 1105601''.328)T - 6''.891\,T + 0''.019\,T \quad (6\text{-}25)$$

$$\Omega = 450160''.280 - (\;\;5^{r} + 482890''.539)T + 7''.455\,T + 0''.008\,T \quad (6\text{-}26)$$

The series is then evaluated term by term, as indicated below.

The first few terms of the series for the nutation in longitude are:

$$\begin{aligned} \Delta\psi = \; & (-17''.1996 - 0''.01742\,T)\sin(\Omega) \\ & +(\;''.2062 + ''.00002\,T)\sin(2\Omega) \\ & +(\;''.0046)\sin(-2l + 2F + \Omega) \\ & +(\;''.0011)\sin(2l - 2F) \\ & \text{etc.} \end{aligned}$$

The first few terms of the series for the nutation in obliquity are:

$$\begin{aligned} \Delta\epsilon = \; & (9''.2025 + 0''.00089\,T)\cos(1\Omega) \\ & +(-\,''.0895 + ''.00005\,T)\cos(2\Omega) \\ & +(-\,''.0024)\cos(-2l + 2F + 1\Omega) \\ & +(\;''.0001)\cos(-2l + 2F + 2\Omega) \\ & \text{etc.} \end{aligned}$$

The nutation in longitude, to be added to longitudes referred to the mean equinox of date, is tabulated to 0".001 for 0^{h} Dynamical Time on each day of the year in the Astronomical Almanac, Section B, in the subsection entitled "Nutation, Obliquity, Day Numbers". The nutation in obliquity, to be added to the mean obliquity of date, is also tabulated there, to the same precision.

The equation of the equinoxes is the right ascension of the mean equinox referred to the true (apparent) equinox. It is equal to $\Delta\psi \cos\epsilon$ and represents the difference between the mean and true (apparent) right ascensions of a heavenly body, in the sense of apparent minus mean. It is thus also equal to the difference between apparent and mean sidereal times. The equation of the equinoxes (EOE) is tabulated to $0^{s}.0001$ in the Almanac, Section B, in the subsection entitled "Universal and Sidereal Times", and is included in the apparent sidereal times given on the same pages.

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6.6.2 Corrections to r.a. and dec. for the effect of nutation.

In the spherical triangle NCP-B-NEP of Figure 6.7, the law of cosines gives

$$\cos(90^\circ-\delta) = \cos\epsilon\cos(90^\circ-\beta) + \sin\epsilon\sin(90^\circ-\beta)\cos(90^\circ-\lambda)$$

$$\sin\delta = \cos\epsilon\sin\beta + \sin\epsilon\sin\lambda\cos\beta \qquad (6\text{-}29)$$

The law of sines applied to the same triangle gives

$$\frac{\sin(90^\circ+\alpha)}{\sin(90^\circ-\beta)} = \frac{\sin(90^\circ-\lambda)}{\sin(90^\circ-\delta)}$$

$$\cos\alpha\cos\delta = \cos\beta\cos\lambda \qquad (6\text{-}30)$$

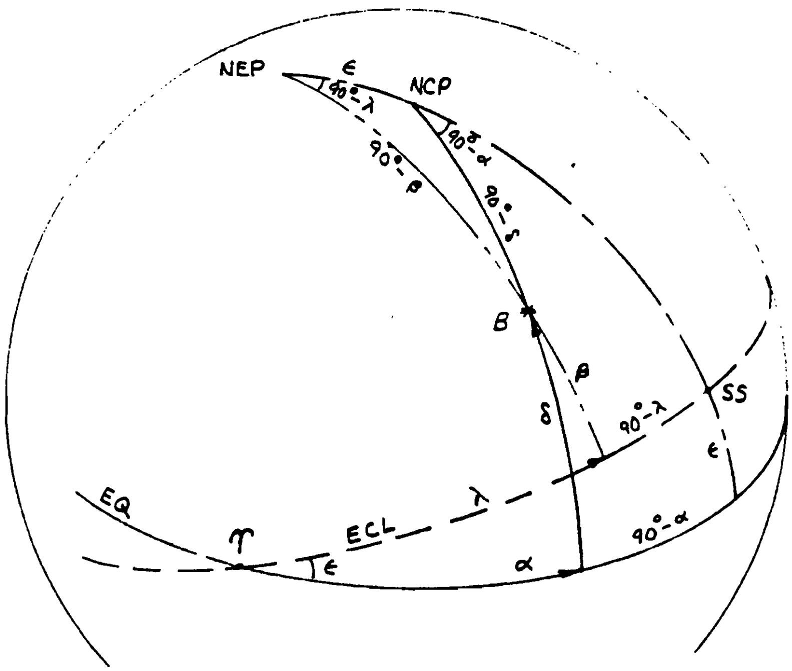
Again in the same triangle, the five-parts formula gives

$$\sin(90^\circ-\delta)\cos(90^\circ+\alpha) = \cos(90^\circ-\beta)\sin\epsilon - \sin(90^\circ-\beta)\cos\epsilon\cos(90^\circ-\lambda)$$

$$-\cos\delta\sin\alpha = \sin\beta\sin\epsilon - \cos\beta\cos\epsilon\sin\lambda$$

$$\sin\alpha\cos\delta = \cos\epsilon\cos\beta\sin\lambda - \sin\epsilon\sin\beta \qquad (6\text{-}31)$$

Fig. 6.7



Differentiating eq. 6-29 with β constant,

$$\cos\delta\, d\delta = -\sin\beta\sin\epsilon\, d\epsilon + \sin\epsilon\cos\beta\cos\lambda\, d\lambda + \cos\beta\sin\lambda\cos\epsilon\, d\epsilon$$

$$= \sin\epsilon\cos\beta\cos\lambda\, d\lambda + (\cos\epsilon\cos\beta\sin\lambda - \sin\epsilon\sin\beta)\, d\epsilon$$

Using eqs. 6-30 and 6-31 in this last expression gives

$$\cos\delta\, d\delta = \sin\epsilon\cos\alpha\cos\delta\, d\lambda + \sin\alpha\cos\delta\, d\epsilon$$

$$d\delta = \sin\epsilon\cos\alpha\, d\lambda + \sin\alpha\, d\epsilon \qquad (6\text{-}32)$$

Now let $d\delta = \Delta\delta_{NU}$, $d\lambda = \Delta\psi$, and $d\epsilon = \Delta\epsilon$; then eq. 6-32 becomes

$$\Delta\delta_{NU} = \delta_{\Upsilon} - \delta_{\bar{\Upsilon}} = \sin\epsilon\cos\alpha\,\Delta\psi + \sin\alpha\,\Delta\epsilon \qquad (6\text{-}33)$$

Differentiating eq. 6-31 with β constant gives

$$-\sin\alpha \sin\delta \, d\delta + \cos\delta \cos\alpha \, d\alpha = \cos\epsilon \cos\beta \cos\lambda \, d\lambda - \cos\beta \sin\lambda \sin\epsilon \, d\epsilon - \sin\beta \cos\epsilon \, d\epsilon$$

$$= \cos\epsilon \cos\beta \cos\lambda \, d\lambda - (\cos\epsilon \sin\beta + \sin\epsilon \cos\beta \sin\lambda) d\epsilon \qquad (6\text{-}34)$$

Using eqs. 6-29 and 6-30 in the right-hand side of the above expression, and moving the term in $d\delta$ across the equal sign results in

$$\cos\delta \cos\alpha \, d\alpha = \cos\epsilon \cos\alpha \cos\delta \, d\lambda - \sin\delta \, d\epsilon + \sin\alpha \sin\delta \, d\delta$$

Substituting eq. 6-32 into this expression gives

$$\cos\delta \cos\alpha \, d\alpha = \cos\epsilon \cos\alpha \cos\delta \, d\lambda - \sin\delta \, d\epsilon + \sin\alpha \sin\delta (\sin\epsilon \cos\alpha \, d\lambda + \sin\alpha \, d\epsilon)$$

Expanding and gathering terms,

$$\cos\delta \cos\alpha \, d\alpha = (\cos\epsilon \cos\alpha \cos\delta + \sin\alpha \sin\delta \sin\epsilon \cos\alpha) d\lambda + (\sin^2\alpha \sin\delta - \sin\delta) d\epsilon$$

$$d\alpha = (\cos\epsilon + \sin\epsilon \sin\alpha \tan\delta) d\lambda + \frac{\tan\delta (\sin^2\alpha - 1) d\epsilon}{\cos\alpha}$$

Using $\sin^2\alpha - 1 = \cos^2\alpha$, $d\alpha = \Delta\alpha_{NU}$, $d\lambda = \Delta\psi$, and $d\epsilon = \Delta\epsilon$,

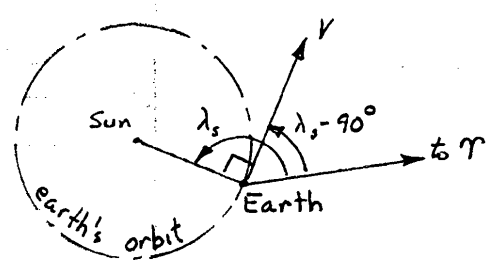
$$\Delta\alpha_{NU} = \alpha_{\Upsilon} - \alpha_{\bar{\Upsilon}} = (\cos\epsilon + \sin\epsilon \sin\alpha \tan\delta) \Delta\psi - \cos\alpha \tan\delta \, \Delta\epsilon \qquad (6\text{-}35)$$

Eqs. 6-35 and 6-33 then give the required corrections to right ascension and declination caused by the nutational components $\Delta\psi$ and $\Delta\epsilon$.

6.7 Annual aberration. As was discussed in section 5.5, there is an apparent shift in position of a star or other heavenly body due to the motion of the observer. In the paragraphs which follow, the effect of this position change due to the motion of the earth in its annual travel around the sun will be discussed. The effect is made up of two parts. The first of these, called the circular annual aberration, is found by considering the orbit of the earth as being circular; the second and smaller part is that due to the actual ellipticity of the earth's orbit, and is known as the e-term of the aberration. The total annual aberration is the sum of the two parts.

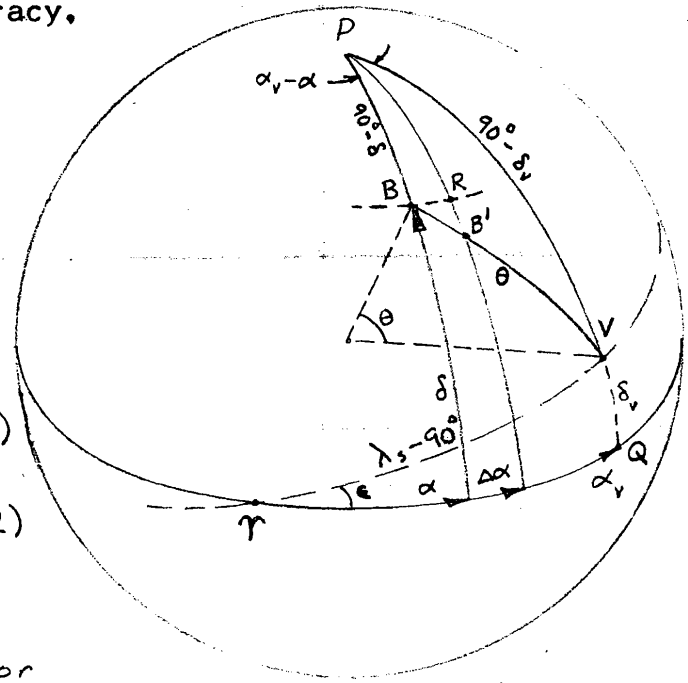
6.7.1 Circular annual aberration. Consider the earth's orbit to be circular, as shown in Fig. 6.8; the instantaneous velocity vector is V, directed at a right angle to the earth-sun line. The ecliptic longitude of the sun is λ_s, so that the vector V points in the direction $\lambda_s - 90^\circ$ from the vernal equinox.

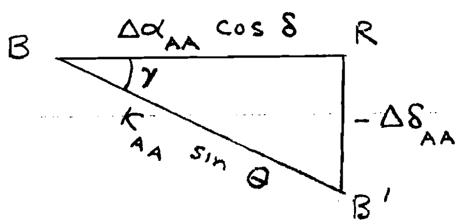
Fig. 6.8



Now consider the celestial sphere as shown in Fig. 6.9; the point labeled V denotes the intersection of the earth's velocity vector with the celestial sphere, and θ is the angle from the body B to the point V. The effect of annual aberration is to shift the apparent position of B to B′, along the arc of θ, ~~by the amount $\Delta\theta = K_{AA} \sin\theta$~~ as was discussed in section 5.5. Since this is a very small displacement, the figure BB′R may be treated, with sufficient accuracy, as a plane triangle.

Fig. 6.9





$$\Delta\alpha_{AA} \cos\delta = K_{AA} \sin\theta \cos\gamma \qquad (6.7\text{-}1)$$

$$-\Delta\delta_{AA} = K_{AA} \sin\theta \sin\gamma \qquad (6.7\text{-}2)$$

In the sph. Δ PBV,

$$\frac{\sin\theta}{\sin(\alpha_v - \alpha)} = \frac{\sin(90^\circ - \delta_v)}{\sin(90^\circ + \gamma)}, \quad \text{or}$$

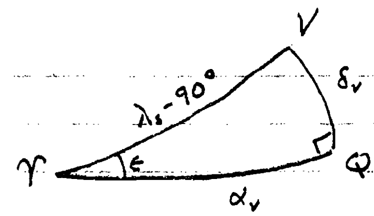
$$\sin\theta \cos\gamma = \cos\delta_v \sin(\alpha_v - \alpha) \qquad (6.7\text{-}3)$$

Also, using the five-parts formula,

$$\sin\theta \cos(90^\circ + \gamma) = \cos(90^\circ - \delta_v) \sin(90^\circ - \delta) - \sin(90^\circ - \delta_v) \cos(90^\circ - \delta) \cos(\alpha_v - \alpha)$$

$$-\sin\theta \sin\gamma = \sin\delta_v \cos\delta - \cos\delta_v \sin\delta \cos(\alpha_v - \alpha) \qquad (6.7\text{-}4)$$

In the right spherical triangle $V Q \Upsilon$,



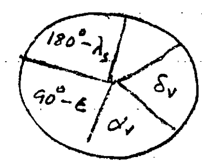
$$\sin(180° - \lambda_s) = \cos\alpha_v \cos\delta_v$$

$$\sin\lambda_s = \cos\alpha_v \cos\delta_v \qquad (6.7\text{-}5)$$

and

$$\sin\delta_v = \cos(180° - \lambda_s)\cos(90° - \epsilon)$$

$$\sin\delta_v = -\cos\lambda_s \sin\epsilon \qquad (6.7\text{-}6)$$



Also, using the five-parts formula,

$$\sin(\lambda_s - 90°)\cos\epsilon = \cos\delta_v \sin\alpha_v - \sin\delta_v \cos\alpha_v \cos 90°$$

$$-\cos\lambda_s \cos\epsilon = \sin\alpha_v \cos\delta_v \qquad (6.7\text{-}7)$$

Now, using (3) in (1),

$$\Delta\alpha_{AA} \cos\delta = K_{AA} \cos\delta_v \sin(\alpha_v - \alpha)$$

$$= K_{AA} \cos\delta_v (\sin\alpha_v \cos\alpha - \cos\alpha_v \sin\alpha)$$

$$= K_{AA} \cos\delta_v \sin\alpha_v \cos\alpha - \cos\delta_v \cos\alpha_v \sin\alpha \qquad (6.7\text{-}8)$$

Using (5) and (7) in (8),

$$\Delta\alpha_{AA} \cos\delta = K_{AA}(-\cos\alpha \cos\lambda_s \cos\epsilon - \sin\alpha \sin\lambda_s)$$

or, after rearranging,

20.″494

$$\Delta\alpha_{CAA} = \alpha' - \alpha = \frac{-K_{AA}}{\cos\delta}\left(\sin\alpha \sin\lambda_{s_{\text{un}}} + \cos\alpha \cos\lambda_s \cos\epsilon\right) \qquad (6.7\text{-}9)$$

Next, using (4) in (2),

$$\Delta\delta_{AA} = K_{AA}\left(\sin\delta_v \cos\delta - \cos\delta_v \sin\delta \cos(\alpha_v - \alpha)\right)$$

$$= K_{AA}\left(\sin\delta_v \cos\delta - \cos\delta_v \sin\delta \left[\cos\alpha_v \cos\alpha + \sin\alpha_v \sin\alpha\right]\right)$$

$$= K_{AA}\left(\sin\delta_v \cos\delta - \cos\delta_v \sin\delta \cos\alpha_v \cos\alpha - \cos\delta_v \sin\delta \sin\alpha_v \sin\alpha\right) \qquad (6.7\text{-}10)$$

Using (5), (6), and (7) in (10) gives

$$\Delta\delta_{AA} = K_{AA}(-\cos\delta \cos\lambda_s \sin\epsilon - \cos\alpha \sin\delta \sin\lambda_s + \sin\alpha \sin\delta \cos\lambda_s \cos\epsilon)$$

$$\Delta\delta_{AA} = \delta' - \delta = -K_{AA}\left(\cos\alpha \sin\delta \sin\lambda_s + \cos\delta \cos\lambda_s \sin\epsilon - \sin\alpha \sin\delta \cos\lambda_s \cos\epsilon\right) \qquad (6.7\text{-}11)$$

Eqs. 9 and 11 then give the corrections to right ascension and declination caused by the circular component of the annual aberration.

6.7.2 The e-term of aberration. The effect of the eccentricity of the earth's orbit on annual aberration is discussed in the paragraphs which follow. It will be shown that the effect is of the order of 0".34 except for close circumpolar stars, hence may usually be neglected.

Consider a body moving about its primary in an elliptical orbit, as shown in Fig. 6.10a. The total velocity vector V is directed tangentially to the ellipse, and may be considered as the resultant of the circular tangential velocity vector $r\dot{\theta}$ and the radial velocity vector $\dot{r}$, as shown in Fig. 6.10b.

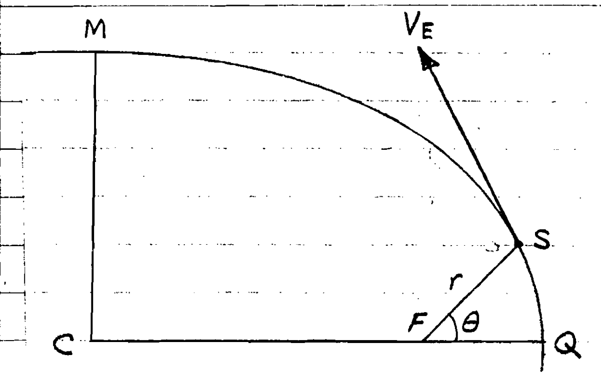


Fig. 6.10a

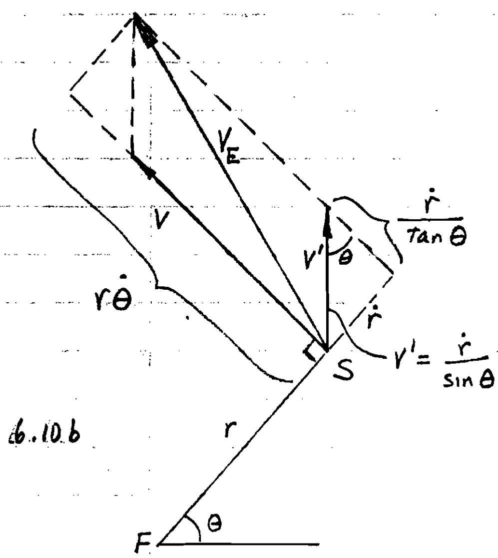


Fig. 6.10b

The equation of the ellipse in polar form is

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \qquad (6.7\text{-}12)$$

Differentiating this with respect to time gives

$$\dot{r} = -\frac{a(1-e^2)}{(1+e\cos\theta)^2}(-e\sin\theta\,\dot{\theta})$$

$$V' = \frac{\dot{r}}{\sin\theta} = \frac{ae(1-e^2)\dot{\theta}}{(1+e\cos\theta)^2} \qquad (6.7\text{-}13)$$

$$\frac{\dot{r}}{\tan\theta} = \frac{\dot{r}}{\sin\theta}\cdot\cos\theta = \frac{ae(1-e^2)\dot{\theta}\cos\theta}{(1+e\cos\theta)^2}$$

$$V = r\dot{\theta} - \frac{\dot{r}}{\tan\theta} = \frac{a(1-e^2)\dot{\theta}}{1+e\cos\theta} - \frac{ae(1-e^2)\dot{\theta}\cos\theta}{(1+e\cos\theta)^2}$$

$$= \frac{a(1-e^2)\dot{\theta}}{1+e\cos\theta}\left(1 - \frac{e\cos\theta}{1+e\cos\theta}\right) = \frac{a(1-e^2)\dot{\theta}}{1+e\cos\theta}\left(\frac{1+e\cos\theta - e\cos\theta}{1+e\cos\theta}\right)$$

$$V = \frac{a(1-e^2)\dot{\theta}}{(1+e\cos\theta)^2} \qquad (6.7-14)$$

Comparing (13) and (14), it is apparent that

$$V' = eV \qquad (6.7-15)$$

It will now be shown that the components V and V' are constant in value.

In Fig. 6.11, the arc MPQ is part of an ellipse with center C, semimajor axis a, seminor axis b, and eccentricity e. The slope of the ellipse at P is along the line PL_E.

$b = a^2 - e^2a^2$

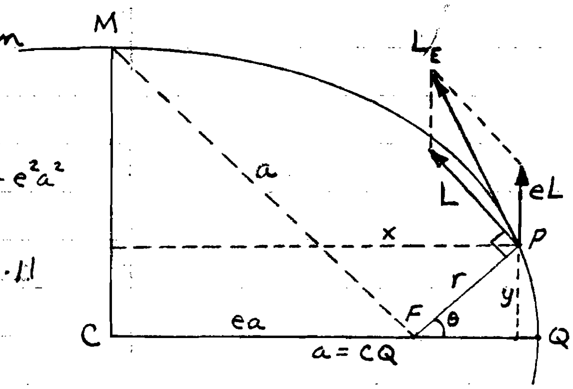
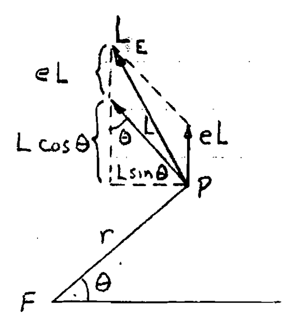


Fig. 6.11

Suppose that L is a line vector which is of constant length, and which is always directed at right angles to the radius vector r, at P; also suppose that at P there is another line vector of constant length eL, directed at right angles to the semimajor axis. The line vector L_E is then the vector sum of L and eL, and the slope of L_E is given by

$$\text{slope} = \frac{dy}{dx} = \frac{eL + L\cos\theta}{-L\sin\theta} = -\frac{e+\cos\theta}{\sin\theta} \qquad (6.7-16)$$



The equation of the ellipse in Cartesian coordinates is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (6.7-17)$$

or

$$b^2x^2 + a^2y^2 = a^2b^2 \qquad (6.7-18)$$

Differentiating (18),

$$b^2 \cdot 2x\,dx + a^2 \cdot 2y\,dy = 0$$

$$\frac{dy}{dx} = -\frac{b^2}{a^2} \cdot \frac{x}{y} \qquad (6.7\text{-}19)$$

From Fig. 6.11,

$$x = ea + r\cos\theta \qquad \text{and} \qquad y = r\sin\theta$$

Substituting these relations in (19),

$$\frac{dy}{dx} = -\frac{b^2}{a^2} \cdot \frac{ea + r\cos\theta}{r\sin\theta} \qquad (6.7\text{-}20)$$

Now using (12) and the relation $b^2 = a^2(1-e^2)$,

$$\frac{dy}{dx} = -\frac{a^2(1-e^2)}{a^2} \cdot \frac{ea + \frac{a(1-e^2)\cos\theta}{1+e\cos\theta}}{\frac{a(1-e^2)}{1+e\cos\theta}\sin\theta}$$

$$\frac{dy}{dx} = -\frac{e + e^2\cos\theta + \cos\theta - e^2\cos\theta}{\sin\theta} = -\frac{e+\cos\theta}{\sin\theta} \qquad (6.7\text{-}21)$$

which is the same result as (16) above, thus showing that the assumption regarding the constancy of the vectors L and eL satisfies the conditions of the ellipse.

The numerical values of V and V′ may be found from Keplerian orbit theory, in which it is shown that

$$\dot{\theta} = \frac{\sqrt{a(1-e^2)\mu}}{r^2} \qquad (6.7\text{-}22)$$

where

$\mu = GM$

$G = 6.673 \times 10^{-11}$ $m^3/kg\text{-}s^2$, the Newtonian gravitational constant

M = mass of the central body in kilograms

Let

$$h^2 = a(1-e^2)\mu \qquad (6.7\text{-}23)$$

Then (12), (14), and (22) become, respectively,

$$r = \frac{h}{\mu(1+e\cos\theta)} \qquad (6.7\text{-}24)$$

$$V = \frac{h\dot{\theta}}{\mu(1+e\cos\theta)^2} \qquad (6.7\text{-}25)$$

$$\dot{\theta} = \frac{h}{r^2} = \frac{\mu^2(1+e\cos\theta)^2}{h^2} \qquad (6.7\text{-}26)$$

Using (26) in (25) gives

$$V = \frac{h}{\mu(1+e\cos\theta)^2} \cdot \frac{\mu^2(1+e\cos\theta)^2}{h}$$

$$V = \frac{\mu}{h} \qquad (6.7\text{-}27)$$

Since μ and h are both constant, then V is constant. Since e is constant, then $V' = eV$ is constant.

For the case of the earth moving around the sun,

$\mu = GM_\odot = 13.246 \times 10^{10}$ km^3/s^2 (from the Explanatory Supplement)

$a = 149{,}600{,}000$ km

$e = 0.01673$

Then $h^2 = 149600000(1-.01673^2)\,13.246 \times 10^{10}$

$h = 4.4509 \times 10^9$ km^4/s^2

$$V = \frac{13.246 \times 10^{10}}{4.451 \times 10^9} = 29.76 \text{ km/s}$$

$V' = eV = .01673(29.76) = 0.50$ km/s

w_s = argument of perigee (small Ω)

The effect of V′ on the right ascension and declination of a body may then be obtained from eqs. (9) and (11) by replacing λ_s with ω_s, and by multiplying by the eccentricity e. The result of these two operations gives

$$\Delta\alpha_{EAA} = \frac{-eK_{AA}}{\cos\delta}\left(\sin\alpha \sin\omega_s + \cos\alpha \cos\omega_s \cos\epsilon\right) \quad (6.7\text{-}28)$$

$$\Delta\delta_{EAA} = -eK_{AA}\left(\cos\alpha \sin\delta \sin\omega_s + \cos\delta \cos\omega_s \sin\epsilon - \sin\alpha \sin\delta \cos\omega_s \cos\epsilon\right) \quad (6.7\text{-}29)$$

Eqs. (28) and (29) then give the corrections to right ascension and declination for the effect of the eccentricity on annual aberration.

At the present time, the longitude of perigee is about 281°, $e \approx 0.01673$, the obliquity of the ecliptic is about 23°.5, and the constant of annual aberration is 20″.496; the maximum values which may be reached by eqs. (28) and (29) are then

$$(\Delta\alpha_{EAA})_{max} \approx \frac{-.01673\,(20''.5)}{\cos\delta}\left(\sin\alpha \sin 281° + \cos\alpha \cos 281° \cos 23°.5\right)$$

$$(\Delta\alpha_{EAA})_{max} \approx \frac{-0''.343}{\cos\delta}\left(-0.982 \sin\alpha + 0.175 \cos\alpha\right) \quad (6.7\text{-}30)$$

and

$$(\Delta\delta_{EAA})_{max} \approx -0.01673\,(20''.5)\left(\cos\alpha \sin\delta \sin 281° + \cos\delta \cos 281° \sin 23°.5 - \sin\alpha \sin\delta \cos 281° \cos 23°.5\right)$$

$$(\Delta\delta_{EAA})_{max} \approx -0''.343\left(-0.982 \cos\alpha \sin\delta + 0.076 \cos\delta - 0.175 \sin\alpha \sin\delta\right) \quad (6.7\text{-}31)$$

Consideration of eqs. (30) and (31) shows that, for stars with declinations from −70° to +70°, the maximum value of the e-term is 1″ or less, hence may usually be ignored.

6.8 Annual parallax.

Consider Fig. 6.8-1 which shows the sun, the earth, and a star; the plane defined by these three bodies is not, in general, the ecliptic plane. The annual parallax of the star is the angle

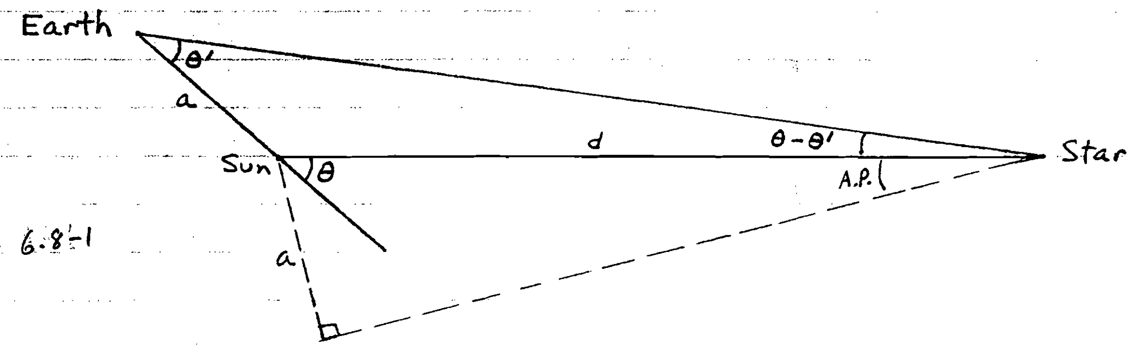


Fig. 6.8-1

at the star subtended by the radius of the earth's orbit, taken as circular (with small error, since the orbital eccentricity is 0.01673). Then

$$\sin \text{A.P.} = \frac{a}{d} \qquad 6.8\text{-}(1)$$

The heliocentric direction of the star is defined by the angle θ, and the geocentric direction is defined by the angle θ'. The angle at the star subtended by the instantaneous radius vector of the earth is $\theta-\theta'$.

From the law of sines applied to the plane triangle Earth-Sun-Star,

$$\frac{\sin(\theta-\theta')}{a} = \frac{\sin\theta'}{d} \qquad 6.8\text{-}(2)$$

and, upon substitution for d from (1),

$$\frac{\sin(\theta-\theta')}{a} = \frac{\sin\theta'}{a/\sin \text{A.P.}}$$

$$\sin(\theta-\theta') = \sin \text{A.P.} \sin\theta' \qquad 6.8\text{-}(3)$$

Annual parallaxes of stars never exceed 1" in value, so that we may write (3) as

$$\theta-\theta' = \text{A.P.} \sin\theta \qquad 6.8\text{-}(4)$$

Fig. 6.8-2 represents the celestial sphere with the sun's geocentric position at S. The heliocentric direction of the star is denoted by the line OB; the geocentric direction by OB′. The arc BB′ is equal to the angle $\theta-\theta'$ as given by (4).

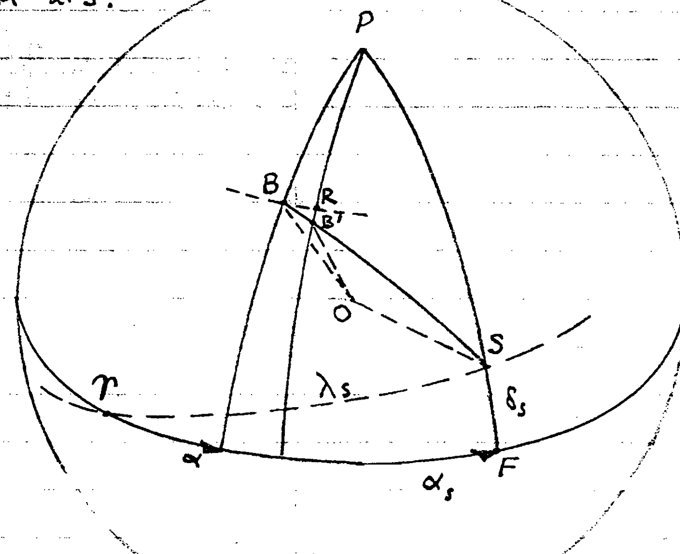


Fig. 6.8-2

The diurnal circle of B cuts the hour circle of B′ at R; the length of the diurnal arc BR is equal to $\Delta\alpha_{AP}\cos\delta$, where $\Delta\alpha_{AP}$ represents the difference in right ascensions of B and B′.

The figure BRB′ is so small that it may, without serious error, be treated as a plane figure, as shown in Fig. 6.8-3. From the figure, and using (4)

$$\Delta\alpha_{AP}\cos\delta = (\theta-\theta')\cos\gamma = A.P.\sin\theta\cos\gamma \quad 6.8\text{-}(5)$$

and

$$-\Delta\delta_{AP} = (\theta-\theta')\sin\gamma = A.P.\sin\theta\sin\gamma \quad 6.8\text{-}(6)$$

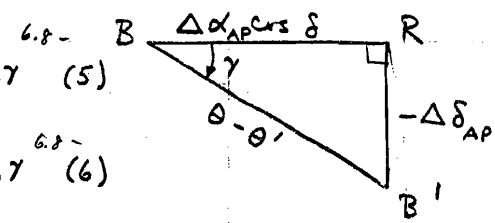
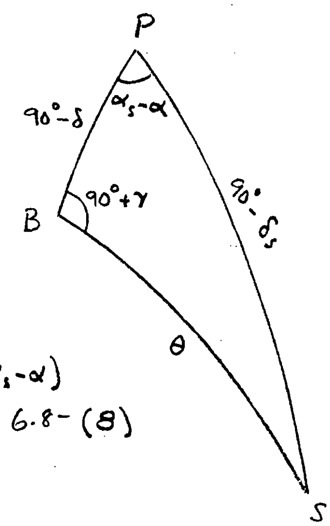


Fig. 6.8-3

In the spherical triangle PBS, using the law of sines,

$$\frac{\sin\theta}{\sin(\alpha_s-\alpha)} = \frac{\sin(90^\circ-\delta_s)}{\sin(90^\circ+\gamma)}$$

$$\sin\theta\cos\gamma = \cos\delta_s\sin(\alpha_s-\alpha) \quad 6.8\text{-}(7)$$



again, using the five-parts formula,

$$\sin\theta\cos(90^\circ+\gamma) = \cos(90^\circ-\delta_s)\sin(90^\circ-\delta) - \sin(90^\circ-\delta_s)\cos(90^\circ-\delta)\cos(\alpha_s-\alpha)$$

$$-\sin\theta\sin\gamma = \sin\delta_s\cos\delta - \cos\delta_s\sin\delta\cos(\alpha_s-\alpha) \quad 6.8\text{-}(8)$$

In the right spherical triangle ϒSF,

$$\frac{\sin \delta_s}{\sin \epsilon} = \frac{\sin \lambda_s}{\sin 90°}$$

$$\sin \delta_s = \sin \lambda_s \sin \epsilon \qquad 6.8-(9)$$

$$\sin(90° - \lambda_s) = \cos \alpha_s \cos \delta_s$$

$$\cos \lambda_s = \cos \alpha_s \cos \delta_s \qquad 6.8-(10)$$

$$\sin \lambda_s \cos \epsilon = \cos \delta_s \sin \alpha_s - \sin \delta_s \cos \alpha_s \cos 90°$$

$$\sin \lambda_s \cos \epsilon = \sin \alpha_s \cos \delta_s \qquad 6.8-(11)$$

Using (7) in (5),

$$\Delta\alpha_{AP} \cos \delta = A.P. \cos \delta_s \sin(\alpha_s - \alpha)$$

$$= A.P. \cos \delta_s (\sin \alpha_s \cos \alpha - \cos \alpha_s \sin \alpha)$$

$$= A.P. (\cos \delta_s \sin \alpha_s \cos \alpha - \cos \delta_s \cos \alpha_s \sin \alpha)$$

Substituting (10) and (11) into this expression yields

$$\Delta\alpha_{AP} = \frac{A.P.}{\cos \delta} (\cos \alpha \sin \lambda_s \cos \epsilon - \sin \alpha \cos \lambda_s) \qquad 6.8-(12)$$

Using (8) in (6),

$$-\Delta\delta_{AP} = -A.P. (\sin \delta_s \cos \delta - \cos \delta_s \sin \delta \cos(\alpha_s - \alpha))$$

$$\Delta\delta_{AP} = A.P. (\sin \delta_s \cos \delta - \cos \delta_s \sin \delta (\cos \alpha_s \cos \alpha + \sin \alpha_s \sin \alpha))$$

$$= A.P. (\sin \delta_s \cos \delta - \cos \delta_s \sin \delta \cos \alpha_s \cos \alpha - \cos \delta_s \sin \delta \sin \alpha_s \sin \alpha)$$

Substituting (9), (10), and (11) in this expression yields

$$\Delta\delta_{AP} = A.P. (\cos \delta \sin \lambda_s \sin \epsilon - \cos \alpha \sin \delta \cos \lambda_s - \sin \alpha \sin \delta \sin \lambda_s \cos \epsilon) \qquad 6.8-(13)$$

Eqs. (12) and (13) then give the required corrections to right ascension and declination caused by annual parallax.

6.9 Reduction from mean to apparent place using Day Numbers.

$$\Delta\alpha_{PR} = \tau\,(m + n \sin\alpha \tan\delta) = \tau n\left(\frac{m}{n} + \sin\alpha \tan\delta\right)$$

$$\Delta\alpha_{NU} = (\cos\epsilon + \sin\epsilon \sin\alpha \tan\delta)\,\Delta\psi - \cos\alpha \tan\delta\,\Delta\epsilon$$

$$= \Delta\psi \cos\epsilon + \Delta\psi \sin\epsilon \sin\alpha \tan\delta - \Delta\epsilon \cos\alpha \tan\delta + \Delta\psi \cdot \frac{m}{n} \sin\epsilon - \Delta\psi \cdot \frac{m}{n} \sin\epsilon$$

$$\Delta\alpha_{NU} = \Delta\psi \sin\epsilon\left(\frac{m}{n} + \sin\alpha \tan\delta\right) + (-\Delta\epsilon)\cos\alpha \tan\delta + \Delta\psi\left(\cos\epsilon - \frac{m}{n}\sin\epsilon\right)$$

$$\Delta\alpha_{AA} = \frac{-K_A}{\cos\delta}(\sin\alpha \sin\lambda_s + \cos\alpha \cos\lambda_s \cos\epsilon) \qquad \text{(neglecting the e-term)}$$

$$= \frac{-K_A}{\cos\delta} \sin\alpha \sin\lambda_s - \frac{K_A}{\cos\delta} \cos\alpha \cos\lambda_s \cos\epsilon$$

$$\Delta\alpha_{AA} = (-K_A \cos\lambda_s \cos\epsilon)\frac{\cos\alpha}{\cos\delta} + (-K_A \sin\lambda_s)\frac{\sin\alpha}{\cos\delta}$$

$$\Delta\alpha_{AP} = \frac{AP}{\cos\delta}(\cos\alpha \sin\lambda_s \cos\epsilon - \sin\alpha \cos\lambda_s)$$

$$= AP\left(\sin\lambda_s \cos\epsilon \frac{\cos\alpha}{\cos\delta} - \cos\lambda_s \frac{\sin\alpha}{\cos\delta}\right)$$

$$= AP\left(Y_\odot \frac{\cos\alpha}{\cos\delta} - X_\odot \frac{\sin\alpha}{\cos\delta}\right)$$

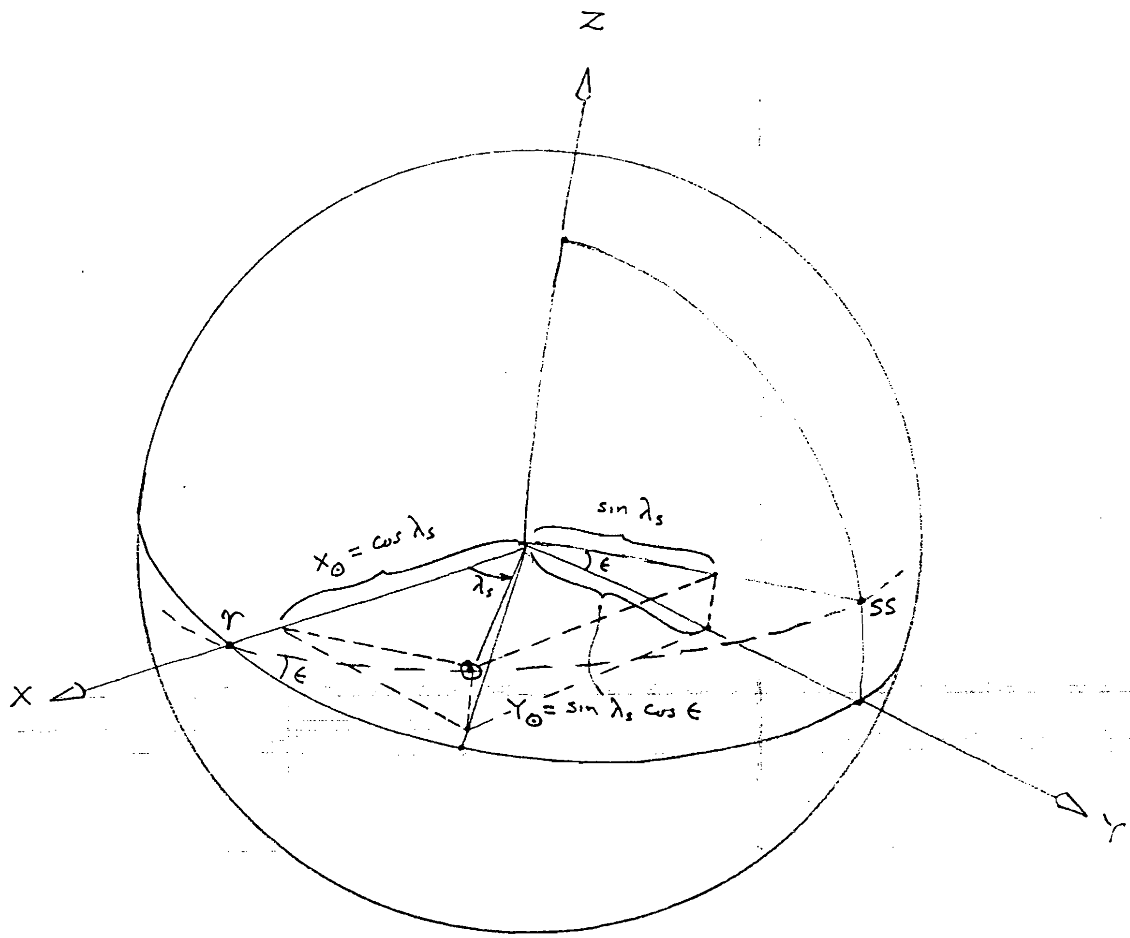
$$\Delta\alpha_{AP} = AP\left(X_\oplus \frac{\sin\alpha}{\cos\delta} - Y_\oplus \frac{\cos\alpha}{\cos\delta}\right) \qquad \text{since } \begin{cases} X_\oplus = -X_\odot \\ Y_\oplus = -Y_\odot \end{cases}$$

Adding, and adding the proper motion term,

$$\Delta\alpha = \Delta\alpha_{PR} + \Delta\alpha_{NU} + \Delta\alpha_{AA} + \Delta\alpha_{AP}$$

$\Delta\alpha = (\tau n + \Delta\psi \sin\epsilon)$	$\left(\frac{m}{n} + \sin\alpha \tan\delta\right)$	$\Delta\alpha = A\,a$
$+ (-\Delta\epsilon)$	$(\cos\alpha \tan\delta)$	$+ B\,b$
$+ (-K_A \cos\lambda_s \cos\epsilon)$	$\left(\frac{\cos\alpha}{\cos\delta}\right)$	$+ C\,c$
$+ (-K_A \sin\lambda_s)$	$\left(\frac{\sin\alpha}{\cos\delta}\right)$	$+ D\,d$
$+ \Delta\psi\left(\cos\epsilon - \frac{m}{n}\sin\epsilon\right)$		$+ E$
$+ AP\left(X \frac{\sin\alpha}{\cos\delta} - Y \frac{\cos\alpha}{\cos\delta}\right)$		$+ AP(dX - cY)$
$+ \tau\mu_\alpha$		$+ \tau\mu_\alpha$

Day #'s — star constants


Z
X
Y
SS
♈
sin λs
Xʘ = cos λs
λs
ε
Yʘ = sin λs cos ε

6.11 The Besselian Day Number method of star place reduction.

$$\alpha = \alpha_1 + \tau\mu_\alpha + Aa + Bb + Cc + Dd + E + J \tan^2 \delta_1 + AP(dX - cY) \qquad (6\text{-}60)$$

$$\delta = \delta_1 + \tau\mu_\delta + Aa' + Bb' + Cc' + Dd' + J' \tan \delta_1 + AP(d'X - c'Y) \qquad (6\text{-}61)$$

where the '1' subscripts denote the mean place at the epoch 19YY.5, which is the middle of the current Julian year.

The Besselian Star Constants are:

$R_{.1}$ – $a = \frac{m}{n} + \sin\alpha_1 \tan\delta_1$

$R_{.2}$ – $b = \cos\alpha_1 \tan\delta_1$

$R_{.3}$ – $c = \cos\alpha_1 / \cos\delta_1$

$R_{.4}$ – $d = \sin\alpha_1 / \cos\delta_1$

$R_{.5}$ – $a' = \cos\alpha_1$

$R_{.6}$ – $b' = -\sin\alpha_1$

$R_{.7}$ – $c' = \tan\bar{\epsilon} \cos\delta_1 - \sin\alpha_1 \sin\delta_1$

$R_{.8}$ – $d' = \cos\alpha_1 \sin\delta_1$

$R_{.9} = \tan\delta_1$

m and n are the annual precession in right ascension and declination at the middle of the Julian year, as given in the Almanac, Section B.

$\bar{\epsilon}$ is the mean obliquity of the ecliptic, again from Section B of the Almanac.

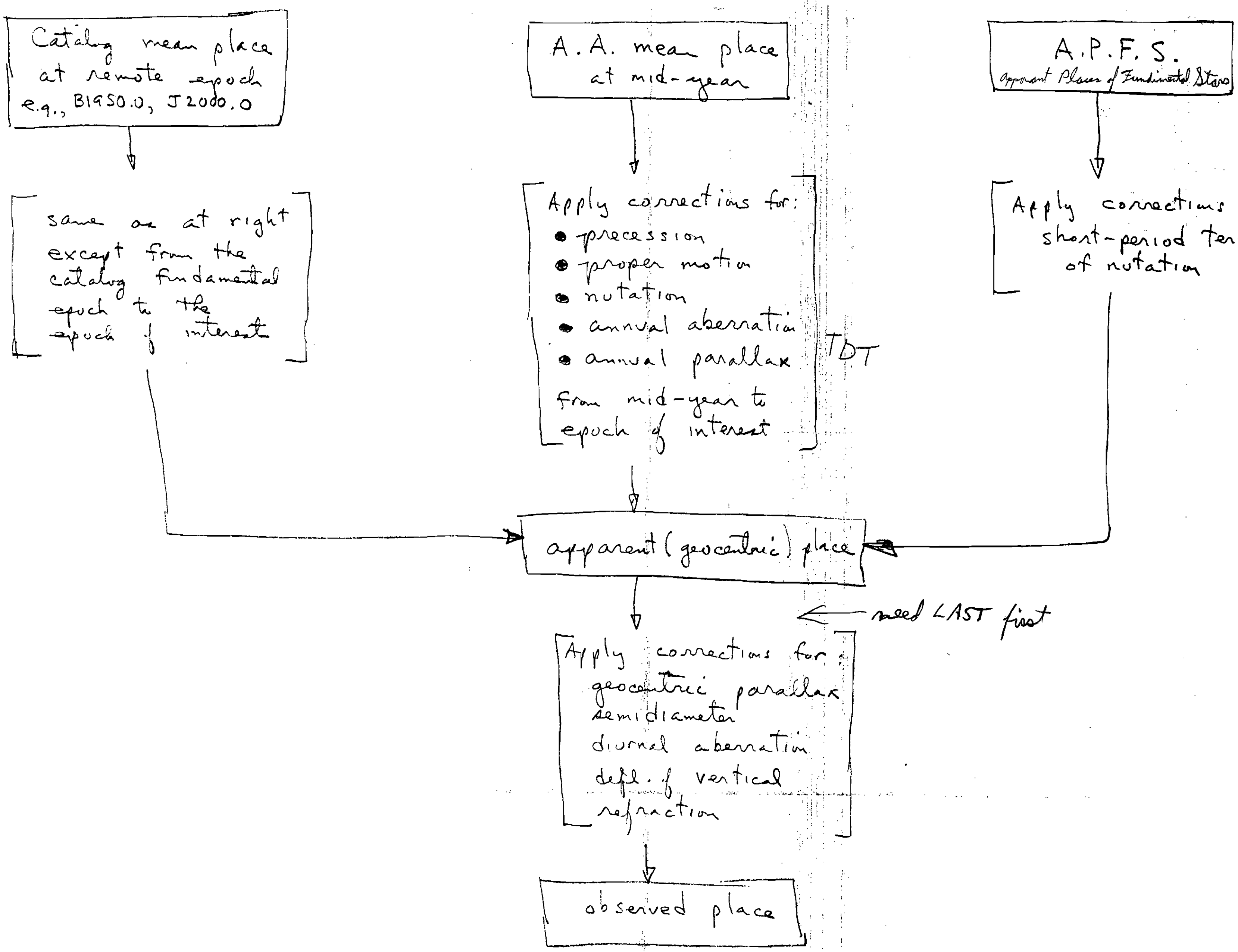
τ denotes the fraction of the Julian year elapsed from the epoch of calculation to the epoch 19YY.5; this definition results in τ being negative during the first half of the year and positive during the last half.

μ denotes the annual proper motion in either right ascension or declination, as indicated by the subscript; values of μ are given in various catalogs, such as the FK4 and the FK5.

A, B, C, D, E, J, and J' are the Besselian Day Numbers; values for each of these are tabulated in Section B of the Almanac. The quantities A, B, and E give the reduction for precession and nutation; C and D give the reduction for annual aberration. The second-order Day Numbers J and J' are so small that they can usually be ignored for the purposes of the present text.

AP is the annual parallax of the star; values of AP are given in various catalogs, such as the Yale "General Catalog of Trigonometric Stellar Parallaxes".

X and Y are the heliocentric equatorial rectangular coordinates of the Earth, referred to the mean equinox and equator at the epoch J2000.0; daily values of X and Y are tabulated in the Almanac, Section B.


Catalog mean place at remote epoch e.g., B1950.0, J2000.0
same as at right except from the catalog fundamental epoch to the epoch of interest
A.A. mean place at mid-year
Apply corrections for:
• precession
• proper motion
• nutation
• annual aberration
• annual parallax
from mid-year to epoch of interest
TDT
A.P.F.S.
Apparent Places of Fundamental Stars
Apply corrections for: short-period terms of nutation
apparent (geocentric) place
need LAST first
Apply corrections for:
geocentric parallax
semidiameter
diurnal aberration
defl. of vertical
refraction
observed place

Exercises

6-1. Find the values of ζ_A , z_A , and θ_A at the epoch J19YY.5, the middle of the current year.

6-2. The J2000.0 mean place of a certain star is 15 hours in right ascension and +30 degrees declination. Find the mean place of this star at the epoch J19YY.5 including only the effect of precession.

6-3. Further reduce the J19YY.5 mean place of the same star to include the effect of proper motion. The centennial proper motions of the star are $+35^s$ in right ascension and $-40''$ in declination.

6-4. Calculate the fundamental arguments required for use with the 1980 IAU Theory of Nutation, for the epoch of problem 6-2.

6-5. Calculate the first three terms of the series for the nutation, in both longitude and obliquity, for the same epoch.

6-6. Using values of $\Delta\psi$ and $\Delta\epsilon$ taken from the Almanac, further reduce the star place from problem 6-3 to include the effect of nutation.

6-7. Further reduce the place of the same star to include the effect of annual aberration.

6-8. Further reduce the place of the same star to include the effect of annual parallax; the annual parallax of the star is $0''.7$.

6-9. Reduce the mean place of the star ϵ Indi (number 8387 in the Bright Star table in the Almanac) considering only the effect of precession, to the epoch February $9^d\, 19^h\, 26^m\, 14^s$ MST.

6-10. Further reduce the mean place of the same star, to the same epoch, to include the effect of proper motion. The centennial values of proper motion, taken from the FK4 catalog, are $+48^s.2$ in right ascension and $-254''.5$ in declination.

6-11. Further reduce the place of the star ϵ Indi to include the effect of nutation.

6-12. Further reduce the place of the star ϵ Indi to include the effect of annual aberration.

6-13. Further reduce the place of the star ϵ Indi to include the effect of annual parallax. The annual parallax of this star is $0''.291$.

APPENDIX B

Excerpts from "THE ASTRONOMICAL ALMANAC - 1985"

and from certain earlier years of

"THE ASTRONOMICAL ALMANAC"

and "THE AMERICAN EPHEMERIS AND NAUTICAL ALMANAC"

Table B-0

UNIVERSAL AND SIDEREAL TIMES, 1983 *

Date 0ʰ U.T.	Julian Date	G. SIDEREAL TIME (G. H. A. of the Equinox) Apparent			Mean	Equation of Equinoxes at 0ʰ U.T.	G. S. D. 0ʰ G. S. T.	U. T. at 0ʰ G. M. S. T. (Greenwich Transit of the Mean Equinox)			
	244	h	m	s	s	s	245		h	m	s
Feb. 15	5380.5	9	37	43.9499	44.9224	-0.9726	2077.0	Feb. 15	14	19	53.8184
16	5381.5	9	41	40.4988	41.4778	.9790	2078.0	16	14	15	57.9089
17	5382.5	9	45	37.0472	38.0332	.9859	2079.0	17	14	12	01.9994
18	5383.5	9	49	33.5962	34.5885	.9923	2080.0	18	14	08	06.0900
19	5384.5	9	53	30.1468	31.1439	.9970	2081.0	19	14	04	10.1805
20	5385.5	9	57	26.6999	27.6993	-0.9993	2082.0	20	14	00	14.2710
21	5386.5	10	01	23.2559	24.2546	.9987	2083.0	21	13	56	18.3616
22	5387.5	10	05	19.8148	20.8100	.9952	2084.0	22	13	52	22.4521
23	5388.5	10	09	16.3757	17.3653	.9896	2085.0	23	13	48	26.5426
24	5389.5	10	13	12.9371	13.9207	.9836	2086.0	24	13	44	30.6332
25	5390.5	10	17	09.4971	10.4761	-0.9789	2087.0	25	13	40	34.7237
26	5391.5	10	21	06.0541	07.0314	.9773	2088.0	26	13	36	38.8142
27	5392.5	10	25	02.6074	03.5868	.9794	2089.0	27	13	32	42.9048
28	5393.5	10	28	59.1574	60.1422	.9848	2090.0	28	13	28	46.9953
Mar. 1	5394.5	10	32	55.7053	56.6975	0.9923	2091.0	Mar. 1	13	24	51.0859
2	5395.5	10	36	52.2528	53.2529	-1.0001	2092.0	2	13	20	55.1764
3	5396.5	10	40	48.8014	49.8083	1.0069	2093.0	3	13	16	59.2669
4	5397.5	10	44	45.3520	46.3636	1.0116	2094.0	4	13	13	03.3575
5	5398.5	10	48	41.9050	42.9190	1.0140	2095.0	5	13	09	07.4480
6	5399.5	10	52	38.4600	39.4744	1.0144	2096.0	6	13	05	11.5385
7	5400.5	10	56	35.0164	36.0297	-1.0133	2097.0	7	13	01	15.6291
8	5401.5	11	00	31.5736	32.5851	1.0115	2098.0	8	12	57	19.7196
9	5402.5	11	04	28.1308	29.1405	1.0097	2099.0	9	12	53	23.8101
10	5403.5	11	08	24.6871	25.6958	1.0087	2100.0	10	12	49	27.9007
11	5404.5	11	12	21.2420	22.2512	1.0092	2101.0	11	12	45	31.9912
12	5405.5	11	16	17.7949	18.8065	-1.0116	2102.0	12	12	41	36.0817
13	5406.5	11	20	14.3459	15.3619	1.0160	2103.0	13	12	37	40.1723
14	5407.5	11	24	10.8948	11.9173	1.0224	2104.0	14	12	33	44.2628
15	5408.5	11	28	07.4424	08.4726	1.0303	2105.0	15	12	29	48.3533
16	5409.5	11	32	03.9892	05.0280	1.0388	2106.0	16	12	25	52.4439
17	5410.5	11	36	00.5365	01.5834	-1.0469	2107.0	17	12	21	56.5344
•18	5411.5	11	39	57.0853	58.1387	1.0534	2108.0	18	12	18	00.6249
19	5412.5	11	43	53.6365	54.6941	1.0576	2109.0	19	12	14	04.7155
20	5413.5	11	47	50.1905	51.2495	1.0589	2110.0	20	12	10	08.8060
21	5414.5	11	51	46.7474	47.8048	1.0574	2111.0	21	12	06	12.8965
22	5415.5	11	55	43.3065	44.3602	-1.0537	2112.0	22	12	02	16.9871
23	5416.5	11	59	39.8664	40.9156	1.0492	2113.0	23	11	58	21.0776
24	5417.5	12	03	36.4255	37.4709	1.0454	2114.0	24	11	54	25.1681
25	5418.5	12	07	32.9824	34.0263	1.0439	2115.0	25	11	50	29.2587
26	5419.5	12	11	29.5360	30.5817	1.0456	2116.0	26	11	46	33.3492
27	5420.5	12	15	26.0864	27.1370	-1.0506	2117.0	27	11	42	37.4397
28	5421.5	12	19	22.6343	23.6924	1.0581	2118.0	28	11	38	41.5303
29	5422.5	12	23	19.1812	20.2477	1.0666	2119.0	29	11	34	45.6208
30	5423.5	12	27	15.7287	16.8031	1.0744	2120.0	30	11	30	49.7113
31	5424.5	12	31	12.2781	13.3585	1.0803	2121.0	31	11	26	53.8019
Apr. 1	5425.5	12	35	08.8300	09.9138	-1.0838	2122.0	Apr. 1	11	22	57.8924
2	5426.5	12	39	05.3843	06.4692	-1.0849	2123.0	2	11	19	01.9829

* From "The Astronomical Almanac - 1983"

TABLE B-1 *

CONVERSION OF TIME TO ARC

m	0ʰ	1ʰ	2ʰ	3ʰ	4ʰ	5ʰ	s	SECONDS	s	"	s	"
	° '	° '	° '	° '	° '	° '	s	' "	s	"	s	"
0	0 00	15 00	30 00	45 00	60 00	75 00	0	0 00	0.00	0.00	0.50	7.50
1	0 15	15 15	30 15	45 15	60 15	75 15	1	0 15	.01	0.15	.51	7.65
2	0 30	15 30	30 30	45 30	60 30	75 30	2	0 30	.02	0.30	.52	7.80
3	0 45	15 45	30 45	45 45	60 45	75 45	3	0 45	.03	0.45	.53	7.95
4	1 00	16 00	31 00	46 00	61 00	76 00	4	1 00	.04	0.60	.54	8.10
5	1 15	16 15	31 15	46 15	61 15	76 15	5	1 15	0.05	0.75	0.55	8.25
6	1 30	16 30	31 30	46 30	61 30	76 30	6	1 30	.06	0.90	.56	8.40
7	1 45	16 45	31 45	46 45	61 45	76 45	7	1 45	.07	1.05	.57	8.55
8	2 00	17 00	32 00	47 00	62 00	77 00	8	2 00	.08	1.20	.58	8.70
9	2 15	17 15	32 15	47 15	62 15	77 15	9	2 15	.09	1.35	.59	8.85
10	2 30	17 30	32 30	47 30	62 30	77 30	10	2 30	0.10	1.50	0.60	9.00
11	2 45	17 45	32 45	47 45	62 45	77 45	11	2 45	.11	1.65	.61	9.15
12	3 00	18 00	33 00	48 00	63 00	78 00	12	3 00	.12	1.80	.62	9.30
13	3 15	18 15	33 15	48 15	63 15	78 15	13	3 15	.13	1.95	.63	9.45
14	3 30	18 30	33 30	48 30	63 30	78 30	14	3 30	.14	2.10	.64	9.60
15	3 45	18 45	33 45	48 45	63 45	78 45	15	3 45	0.15	2.25	0.65	9.75
16	4 00	19 00	34 00	49 00	64 00	79 00	16	4 00	.16	2.40	.66	9.90
17	4 15	19 15	34 15	49 15	64 15	79 15	17	4 15	.17	2.55	.67	10.05
18	4 30	19 30	34 30	49 30	64 30	79 30	18	4 30	.18	2.70	.68	10.20
19	4 45	19 45	34 45	49 45	64 45	79 45	19	4 45	.19	2.85	.69	10.35
20	5 00	20 00	35 00	50 00	65 00	80 00	20	5 00	0.20	3.00	0.70	10.50
21	5 15	20 15	35 15	50 15	65 15	80 15	21	5 15	.21	3.15	.71	10.65
22	5 30	20 30	35 30	50 30	65 30	80 30	22	5 30	.22	3.30	.72	10.80
23	5 45	20 45	35 45	50 45	65 45	80 45	23	5 45	.23	3.45	.73	10.95
24	6 00	21 00	36 00	51 00	66 00	81 00	24	6 00	.24	3.60	.74	11.10
25	6 15	21 15	36 15	51 15	66 15	81 15	25	6 15	0.25	3.75	0.75	11.25
26	6 30	21 30	36 30	51 30	66 30	81 30	26	6 30	.26	3.90	.76	11.40
27	6 45	21 45	36 45	51 45	66 45	81 45	27	6 45	.27	4.05	.77	11.55
28	7 00	22 00	37 00	52 00	67 00	82 00	28	7 00	.28	4.20	.78	11.70
29	7 15	22 15	37 15	52 15	67 15	82 15	29	7 15	.29	4.35	.79	11.85
30	7 30	22 30	37 30	52 30	67 30	82 30	30	7 30	0.30	4.50	0.80	12.00
31	7 45	22 45	37 45	52 45	67 45	82 45	31	7 45	.31	4.65	.81	12.15
32	8 00	23 00	38 00	53 00	68 00	83 00	32	8 00	.32	4.80	.82	12.30
33	8 15	23 15	38 15	53 15	68 15	83 15	33	8 15	.33	4.95	.83	12.45
34	8 30	23 30	38 30	53 30	68 30	83 30	34	8 30	.34	5.10	.84	12.60
35	8 45	23 45	38 45	53 45	68 45	83 45	35	8 45	0.35	5.25	0.85	12.75
36	9 00	24 00	39 00	54 00	69 00	84 00	36	9 00	.36	5.40	.86	12.90
37	9 15	24 15	39 15	54 15	69 15	84 15	37	9 15	.37	5.55	.87	13.05
38	9 30	24 30	39 30	54 30	69 30	84 30	38	9 30	.38	5.70	.88	13.20
39	9 45	24 45	39 45	54 45	69 45	84 45	39	9 45	.39	5.85	.89	13.35
40	10 00	25 00	40 00	55 00	70 00	85 00	40	10 00	0.40	6.00	0.90	13.50
41	10 15	25 15	40 15	55 15	70 15	85 15	41	10 15	.41	6.15	.91	13.65
42	10 30	25 30	40 30	55 30	70 30	85 30	42	10 30	.42	6.30	.92	13.80
43	10 45	25 45	40 45	55 45	70 45	85 45	43	10 45	.43	6.45	.93	13.95
44	11 00	26 00	41 00	56 00	71 00	86 00	44	11 00	.44	6.60	.94	14.10
45	11 15	26 15	41 15	56 15	71 15	86 15	45	11 15	0.45	6.75	0.95	14.25
46	11 30	26 30	41 30	56 30	71 30	86 30	46	11 30	.46	6.90	.96	14.40
47	11 45	26 45	41 45	56 45	71 45	86 45	47	11 45	.47	7.05	.97	14.55
48	12 00	27 00	42 00	57 00	72 00	87 00	48	12 00	.48	7.20	.98	14.70
49	12 15	27 15	42 15	57 15	72 15	87 15	49	12 15	.49	7.35	0.99	14.85
50	12 30	27 30	42 30	57 30	72 30	87 30	50	12 30	0.50	7.50	1.00	15.00
51	12 45	27 45	42 45	57 45	72 45	87 45	51	12 45				
52	13 00	28 00	43 00	58 00	73 00	88 00	52	13 00				
53	13 15	28 15	43 15	58 15	73 15	88 15	53	13 15				
54	13 30	28 30	43 30	58 30	73 30	88 30	54	13 30				
55	13 45	28 45	43 45	58 45	73 45	88 45	55	13 45				
56	14 00	29 00	44 00	59 00	74 00	89 00	56	14 00				
57	14 15	29 15	44 15	59 15	74 15	89 15	57	14 15				
58	14 30	29 30	44 30	59 30	74 30	89 30	58	14 30				
59	14 45	29 45	44 45	59 45	74 45	89 45	59	14 45				

6ʰ = 90°

12ʰ = 180°

18ʰ = 270°

* From "The American Ephemeris and Nautical Almanac - 1980"

TABLE B-2 *

CONVERSION OF ARC TO TIME

DEGREES						MINUTES		SECONDS					
°	h m	°	h m	°	h m	′	m s	″	s	″	s	″	s
0	0 00	60	4 00	120	8 00	0	0 00	0	0.000	0.00	0.000	0.50	0.033
1	0 04	61	4 04	121	8 04	1	0 04	1	0.067	.01	.001	.51	.034
2	0 08	62	4 08	122	8 08	2	0 08	2	0.133	.02	.001	.52	.035
3	0 12	63	4 12	123	8 12	3	0 12	3	0.200	.03	.002	.53	.035
4	0 16	64	4 16	124	8 16	4	0 16	4	0.267	.04	.003	.54	.036
5	0 20	65	4 20	125	8 20	5	0 20	5	0.333	0.05	0.003	0.55	0.037
6	0 24	66	4 24	126	8 24	6	0 24	6	0.400	.06	.004	.56	.037
7	0 28	67	4 28	127	8 28	7	0 28	7	0.467	.07	.005	.57	.038
8	0 32	68	4 32	128	8 32	8	0 32	8	0.533	.08	.005	.58	.039
9	0 36	69	4 36	129	8 36	9	0 36	9	0.600	.09	.006	.59	.039
10	0 40	70	4 40	130	8 40	10	0 40	10	0.667	0.10	0.007	0.60	0.040
11	0 44	71	4 44	131	8 44	11	0 44	11	0.733	.11	.007	.61	.041
12	0 48	72	4 48	132	8 48	12	0 48	12	0.800	.12	.008	.62	.041
13	0 52	73	4 52	133	8 52	13	0 52	13	0.867	.13	.009	.63	.042
14	0 56	74	4 56	134	8 56	14	0 56	14	0.933	.14	.009	.64	.043
15	1 00	75	5 00	135	9 00	15	1 00	15	1.000	0.15	0.010	0.65	0.043
16	1 04	76	5 04	136	9 04	16	1 04	16	1.067	.16	.011	.66	.044
17	1 08	77	5 08	137	9 08	17	1 08	17	1.133	.17	.011	.67	.045
18	1 12	78	5 12	138	9 12	18	1 12	18	1.200	.18	.012	.68	.045
19	1 16	79	5 16	139	9 16	19	1 16	19	1.267	.19	.013	.69	.046
20	1 20	80	5 20	140	9 20	20	1 20	20	1.333	0.20	0.013	0.70	0.047
21	1 24	81	5 24	141	9 24	21	1 24	21	1.400	.21	.014	.71	.047
22	1 28	82	5 28	142	9 28	22	1 28	22	1.467	.22	.015	.72	.048
23	1 32	83	5 32	143	9 32	23	1 32	23	1.533	.23	.015	.73	.049
24	1 36	84	5 36	144	9 36	24	1 36	24	1.600	.24	.016	.74	.049
25	1 40	85	5 40	145	9 40	25	1 40	25	1.667	0.25	0.017	0.75	0.050
26	1 44	86	5 44	146	9 44	26	1 44	26	1.733	.26	.017	.76	.051
27	1 48	87	5 48	147	9 48	27	1 48	27	1.800	.27	.018	.77	.051
28	1 52	88	5 52	148	9 52	28	1 52	28	1.867	.28	.019	.78	.052
29	1 56	89	5 56	149	9 56	29	1 56	29	1.933	.29	.019	.79	.053
30	2 00	90	6 00	150	10 00	30	2 00	30	2.000	0.30	0.020	0.80	0.053
31	2 04	91	6 04	151	10 04	31	2 04	31	2.067	.31	.021	.81	.054
32	2 08	92	6 08	152	10 08	32	2 08	32	2.133	.32	.021	.82	.055
33	2 12	93	6 12	153	10 12	33	2 12	33	2.200	.33	.022	.83	.055
34	2 16	94	6 16	154	10 16	34	2 16	34	2.267	.34	.023	.84	.056
35	2 20	95	6 20	155	10 20	35	2 20	35	2.333	0.35	0.023	0.85	0.057
36	2 24	96	6 24	156	10 24	36	2 24	36	2.400	.36	.024	.86	.057
37	2 28	97	6 28	157	10 28	37	2 28	37	2.467	.37	.025	.87	.058
38	2 32	98	6 32	158	10 32	38	2 32	38	2.533	.38	.025	.88	.059
39	2 36	99	6 36	159	10 36	39	2 36	39	2.600	.39	.026	.89	.059
40	2 40	100	6 40	160	10 40	40	2 40	40	2.667	0.40	0.027	0.90	0.060
41	2 44	101	6 44	161	10 44	41	2 44	41	2.733	.41	.027	.91	.061
42	2 48	102	6 48	162	10 48	42	2 48	42	2.800	.42	.028	.92	.061
43	2 52	103	6 52	163	10 52	43	2 52	43	2.867	.43	.029	.93	.062
44	2 56	104	6 56	164	10 56	44	2 56	44	2.933	.44	.029	.94	.063
45	3 00	105	7 00	165	11 00	45	3 00	45	3.000	0.45	0.030	0.95	0.063
46	3 04	106	7 04	166	11 04	46	3 04	46	3.067	.46	.031	.96	.064
47	3 08	107	7 08	167	11 08	47	3 08	47	3.133	.47	.031	.97	.065
48	3 12	108	7 12	168	11 12	48	3 12	48	3.200	.48	.032	.98	.065
49	3 16	109	7 16	169	11 16	49	3 16	49	3.267	.49	.033	0.99	.066
50	3 20	110	7 20	170	11 20	50	3 20	50	3.333	0.50	0.033	1.00	0.067
51	3 24	111	7 24	171	11 24	51	3 24	51	3.400				
52	3 28	112	7 28	172	11 28	52	3 28	52	3.467				
53	3 32	113	7 32	173	11 32	53	3 32	53	3.533				
54	3 36	114	7 36	174	11 36	54	3 36	54	3.600				
55	3 40	115	7 40	175	11 40	55	3 40	55	3.667				
56	3 44	116	7 44	176	11 44	56	3 44	56	3.733				
57	3 48	117	7 48	177	11 48	57	3 48	57	3.800				
58	3 52	118	7 52	178	11 52	58	3 52	58	3.867				
59	3 56	119	7 56	179	11 56	59	3 56	59	3.933				

90° = 6^h

180° = 12^h

270° = 18^h

* From "The American Ephemeris and Nautical Almanac - 1980"

TABLE B-3

JULIAN DAY NUMBER *

DAYS ELAPSED AT GREENWICH NOON, A. D. 1900–1950

Year	Jan. 0	Feb. 0	Mar. 0	Apr. 0	May 0	June 0	July 0	Aug. 0	Sept. 0	Oct. 0	Nov. 0	Dec. 0
1900	241 5020	5051	5079	5110	5140	5171	5201	5232	5263	5293	5324	5354
1901	5385	5416	5444	5475	5505	5536	5566	5597	5628	5658	5689	5719
1902	5750	5781	5809	5840	5870	5901	5931	5962	5993	6023	6054	6084
1903	6115	6146	6174	6205	6235	6266	6296	6327	6358	6388	6419	6449
1904	6480	6511	6540	6571	6601	6632	6662	6693	6724	6754	6785	6815
1905	241 6846	6877	6905	6936	6966	6997	7027	7058	7089	7119	7150	7180
1906	7211	7242	7270	7301	7331	7362	7392	7423	7454	7484	7515	7545
1907	7576	7607	7635	7666	7696	7727	7757	7788	7819	7849	7880	7910
1908	7941	7972	8001	8032	8062	8093	8123	8154	8185	8215	8246	8276
1909	8307	8338	8366	8397	8427	8458	8488	8519	8550	8580	8611	8641
1910	241 8672	8703	8731	8762	8792	8823	8853	8884	8915	8945	8976	9006
1911	9037	9068	9096	9127	9157	9188	9218	9249	9280	9310	9341	9371
1912	9402	9433	9462	9493	9523	9554	9584	9615	9646	9676	9707	9737
1913	9768	9799	9827	9858	9888	9919	9949	9980	*0011	*0041	*0072	*0102
1914	242 0133	0164	0192	0223	0253	0284	0314	0345	0376	0406	0437	0467
1915	242 0498	0529	0557	0588	0618	0649	0679	0710	0741	0771	0802	0832
1916	0863	0894	0923	0954	0984	1015	1045	1076	1107	1137	1168	1198
1917	1229	1260	1288	1319	1349	1380	1410	1441	1472	1502	1533	1563
1918	1594	1625	1653	1684	1714	1745	1775	1806	1837	1867	1898	1928
1919	1959	1990	2018	2049	2079	2110	2140	2171	2202	2232	2263	2293
1920	242 2324	2355	2384	2415	2445	2476	2506	2537	2568	2598	2629	2659
1921	2690	2721	2749	2780	2810	2841	2871	2902	2933	2963	2994	3024
1922	3055	3086	3114	3145	3175	3206	3236	3267	3298	3328	3359	3389
1923	3420	3451	3479	3510	3540	3571	3601	3632	3663	3693	3724	3754
1924	3785	3816	3845	3876	3906	3937	3967	3998	4029	4059	4090	4120
1925	242 4151	4182	4210	4241	4271	4302	4332	4363	4394	4424	4455	4485
1926	4516	4547	4575	4606	4636	4667	4697	4728	4759	4789	4820	4850
1927	4881	4912	4940	4971	5001	5032	5062	5093	5124	5154	5185	5215
1928	5246	5277	5306	5337	5367	5398	5428	5459	5490	5520	5551	5581
1929	5612	5643	5671	5702	5732	5763	5793	5824	5855	5885	5916	5946
1930	242 5977	6008	6036	6067	6097	6128	6158	6189	6220	6250	6281	6311
1931	6342	6373	6401	6432	6462	6493	6523	6554	6585	6615	6646	6676
1932	6707	6738	6767	6798	6828	6859	6889	6920	6951	6981	7012	7042
1933	7073	7104	7132	7163	7193	7224	7254	7285	7316	7346	7377	7407
1934	7438	7469	7497	7528	7558	7589	7619	7650	7681	7711	7742	7772
1935	242 7803	7834	7862	7893	7923	7954	7984	8015	8046	8076	8107	8137
1936	8168	8199	8228	8259	8289	8320	8350	8381	8412	8442	8473	8503
1937	8534	8565	8593	8624	8654	8685	8715	8746	8777	8807	8838	8868
1938	8899	8930	8958	8989	9019	9050	9080	9111	9142	9172	9203	9233
1939	9264	9295	9323	9354	9384	9415	9445	9476	9507	9537	9568	9598
1940	242 9629	9660	9689	9720	9750	9781	9811	9842	9873	9903	9934	9964
1941	9995	*0026	*0054	*0085	*0115	*0146	*0176	*0207	*0238	*0268	*0299	*0329
1942	243 0360	0391	0419	0450	0480	0511	0541	0572	0603	0633	0664	0694
1943	0725	0756	0784	0815	0845	0876	0906	0937	0968	0998	1029	1059
1944	1090	1121	1150	1181	1211	1242	1272	1303	1334	1364	1395	1425
1945	243 1456	1487	1515	1546	1576	1607	1637	1668	1699	1729	1760	1790
1946	1821	1852	1880	1911	1941	1972	2002	2033	2064	2094	2125	2155
1947	2186	2217	2245	2276	2306	2337	2367	2398	2429	2459	2490	2520
1948	2551	2582	2611	2642	2672	2703	2733	2764	2795	2825	2856	2886
1949	2917	2948	2976	3007	3037	3068	3098	3129	3160	3190	3221	3251
1950	243 3282	3313	3341	3372	3402	3433	3463	3494	3525	3555	3586	3616

* From "The American Ephemeris and Nautical Almanac - 1980"

TABLE B-4

JULIAN DAY NUMBER *

DAYS ELAPSED AT GREENWICH NOON, A. D. 1950–2000

Year	Jan. 0	Feb. 0	Mar. 0	Apr. 0	May 0	June 0	July 0	Aug. 0	Sept. 0	Oct. 0	Nov. 0	Dec. 0
1950	243 3282	3313	3341	3372	3402	3433	3463	3494	3525	3555	3586	3616
1951	3647	3678	3706	3737	3767	3798	3828	3859	3890	3920	3951	3981
1952	4012	4043	4072	4103	4133	4164	4194	4225	4256	4286	4317	4347
1953	4378	4409	4437	4468	4498	4529	4559	4590	4621	4651	4682	4712
1954	4743	4774	4802	4833	4863	4894	4924	4955	4986	5016	5047	5077
1955	243 5108	5139	5167	5198	5228	5259	5289	5320	5351	5381	5412	5442
1956	5473	5504	5533	5564	5594	5625	5655	5686	5717	5747	5778	5808
1957	5839	5870	5898	5929	5959	5990	6020	6051	6082	6112	6143	6173
1958	6204	6235	6263	6294	6324	6355	6385	6416	6447	6477	6508	6538
1959	6569	6600	6628	6659	6689	6720	6750	6781	6812	6842	6873	6903
1960	243 6934	6965	6994	7025	7055	7086	7116	7147	7178	7208	7239	7269
1961	7300	7331	7359	7390	7420	7451	7481	7512	7543	7573	7604	7634
1962	7665	7696	7724	7755	7785	7816	7846	7877	7908	7938	7969	7999
1963	8030	8061	8089	8120	8150	8181	8211	8242	8273	8303	8334	8364
1964	8395	8426	8455	8486	8516	8547	8577	8608	8639	8669	8700	8730
1965	243 8761	8792	8820	8851	8881	8912	8942	8973	9004	9034	9065	9095
1966	9126	9157	9185	9216	9246	9277	9307	9338	9369	9399	9430	9460
1967	9491	9522	9550	9581	9611	9642	9672	9703	9734	9764	9795	9825
1968	9856	9887	9916	9947	9977	*0008	*0038	*0069	*0100	*0130	*0161	*0191
1969	244 0222	0253	0281	0312	0342	0373	0403	0434	0465	0495	0526	0556
1970	244 0587	0618	0646	0677	0707	0738	0768	0799	0830	0860	0891	0921
1971	0952	0983	1011	1042	1072	1103	1133	1164	1195	1225	1256	1286
1972	1317	1348	1377	1408	1438	1469	1499	1530	1561	1591	1622	1652
1973	1683	1714	1742	1773	1803	1834	1864	1895	1926	1956	1987	2017
1974	2048	2079	2107	2138	2168	2199	2229	2260	2291	2321	2352	2382
1975	244 2413	2444	2472	2503	2533	2564	2594	2625	2656	2686	2717	2747
1976	2778	2809	2838	2869	2899	2930	2960	2991	3022	3052	3083	3113
1977	3144	3175	3203	3234	3264	3295	3325	3356	3387	3417	3448	3478
1978	3509	3540	3568	3599	3629	3660	3690	3721	3752	3782	3813	3843
1979	3874	3905	3933	3964	3994	4025	4055	4086	4117	4147	4178	4208
1980	244 4239	4270	4299	4330	4360	4391	4421	4452	4483	4513	4544	4574
1981	4605	4636	4664	4695	4725	4756	4786	4817	4848	4878	4909	4939
1982	4970	5001	5029	5060	5090	5121	5151	5182	5213	5243	5274	5304
1983	5335	5366	5394	5425	5455	5486	5516	5547	5578	5608	5639	5669
1984	5700	5731	5760	5791	5821	5852	5882	5913	5944	5974	6005	6035
1985	244 6066	6097	6125	6156	6186	6217	6247	6278	6309	6339	6370	6400
1986	6431	6462	6490	6521	6551	6582	6612	6643	6674	6704	6735	6765
1987	6796	6827	6855	6886	6916	6947	6977	7008	7039	7069	7100	7130
1988	7161	7192	7221	7252	7282	7313	7343	7374	7405	7435	7466	7496
1989	7527	7558	7586	7617	7647	7678	7708	7739	7770	7800	7831	7861
1990	244 7892	7923	7951	7982	8012	8043	8073	8104	8135	8165	8196	8226
1991	8257	8288	8316	8347	8377	8408	8438	8469	8500	8530	8561	8591
1992	8622	8653	8682	8713	8743	8774	8804	8835	8866	8896	8927	8957
1993	8988	9019	9047	9078	9108	9139	9169	9200	9231	9261	9292	9322
1994	9353	9384	9412	9443	9473	9504	9534	9565	9596	9626	9657	9687
1995	244 9718	9749	9777	9808	9838	9869	9899	9930	9961	9991	*0022	*0052
1996	245 0083	0114	0143	0174	0204	0235	0265	0296	0327	0357	0388	0418
1997	0449	0480	0508	0539	0569	0600	0630	0661	0692	0722	0753	0783
1998	0814	0845	0873	0904	0934	0965	0995	1026	1057	1087	1118	1148
1999	1179	1210	1238	1269	1299	1330	1360	1391	1422	1452	1483	1513
2000	245 1544	1575	1604	1635	1665	1696	1726	1757	1788	1818	1849	1879

* From "The American Ephemeris and Nautical Almanac - 1980"

Table B-5 *

OPTICAL OBSERVATORIES, 1981

Place	Instrument	West Longitude	Latitude	Height (Sea Level)	Height (Geoid. Corr.)
		° ′ ″	° ′ ″	m	m
Flagstaff, Arizona Flagstaff Station U.S. Naval Observatory P.O. Box 1149 Flagstaff, Arizona 86002	61-in Astrometric Reflector	+111 44 23.6	+35 11 02.5	2316	− 7
	40-in Ritchey-Chrétien Reflector	+111 44 14.9	+35 11 03.6	2312	− 7
	24-in Cassegrain Reflector	+111 44 15.5	+35 11 04.5	2313	− 7
Flagstaff, Arizona Lowell Observatory P.O. Box 1269 Flagstaff, Arizona 86001	72-in Perkins Reflector	+111 32 09.3	+35 05 48.6	2198	− 7
	42-in Ritchey-Chrétien Reflector	+111 32 08	+35 05 48	2198	− 7
	24-in Clark Equatorial Refractor	+111 39 48	+35 12 08	2210	− 7
	24-in Morgan Reflector	+111 39 54	+35 12 14	2204	− 7
	13-in Lowell Refractor	+111 32 08	+35 05 44	2200	− 7
	42-in Clark Reflector	+111 32 09.30	+35 05 46.6	2180	− 7
	31-in Reflector	+111 32 09	+35 05 55	2198	− 7
Glasgow, Scotland Glasgow University Observatory Acre Road/Maryhill Road Glasgow G20 0TL, Scotland	0.5-m Ritchey-Chrétien Reflector	+ 4 18 20	+55 54 08	53	
	0.3-m Telescope	+ 4 18 22	+55 54 09	53	
	6-cm Transit Telescope	+ 4 18 19	+55 54 08	53	
Greenbelt, Maryland Goddard Research Observatory NASA/Goddard Space Flight Center Greenbelt, Maryland 20771	91.5-cm Cass./Coudé Reflector	+ 76 49 37.14	+39 01 15.9	53	+ 1
	48-in Coudé Telescope	+ 76 49 43.33	+39 01 16.9	53	+ 1
	2-Element Interferometer	+ 76 49 33	+39 01 17	44	
Harvard, Massachusetts George R. Agassiz Station Harvard College Observatory Harvard, Massachusetts 01451	155-cm Wyeth Fecker Reflector	+ 71 33 29.32	+42 30 19.0	185	+ 6
	80-ft Equatorial Radio Antenna	+ 71 33 30	+42 30 13	183	
Herstmonceux, England Royal Greenwich Observatory Herstmonceux Castle Hailsham, East Sussex BN27 1RP England	Cooke Transit Circle	− 0 20 15.45	+50 52 18	34	
	98-in Isaac Newton Reflector	− 0 20 46.5	+50 51 58	53	
	25-cm Photographic Zenith Tube	− 0 20 19.5	+50 52 19	28	
	26-in Thompson Refractor	− 0 20 48.0	+50 52 09	50	
Hobart, Tasmania University of Tasmania Observatory G.P.O. Box 2520 Hobart, Tasmania 7001 Australia	1-m Optical Telescope	−147 32 00	−42 50 00	300	
	0.4-m Optical Telescope	−147 32 00	−42 50 00	300	
	256x256-m Array	−147 32 00	−42 50 00	300	
	512x512-m Array	−147 32 00	−42 50 00	300	
Hoher List, Germany F.R. Hoher List Observatory University of Bonn D-5568 Daun, (Eifel) Germany F.R.	106-cm Cassegrain Telescope	− 6 51 00.02	+50 09 45.3	533	
	36-cm Cassegrain Telescope	− 6 50 59.05	+50 09 47.5	543	
	34-cm Schmidt Telescope	− 6 50 56.57	+50 09 46.8	545	
	36/30-cm Refractor	− 6 51 02.02	+50 09 48.5	541	
	30-cm Astrograph	− 6 50 58.25	+50 09 47.5	544	
Jungfraujoch, Switzerland High Alpine Research Station Sidlerstrasse 5 3012 Berne, Switzerland	76-cm Cass./Coudé Telescope	− 7 59 06	+46 32 53	3576	
Kamogata-Cho, Japan Okayama Astrophysical Observatory Kamogata Cho, Asakuchi-Gun Okayama-Ken, 719-02 Japan	188-cm Reflector	−133 35 47.29	+34 34 26.1	372	−18
	91-cm Reflector	−133 35 46.6	+34 34 22.8	365	−18
	64-cm Solar Reflector	−133 35 46	+34 34 18	350	−18

* From "The Astronomical Almanac - 1981"

APPENDIX C

MISCELLANEOUS TABLES AND CHARTS

Table C-2 LIST OF RADIO TIME SIGNALS*

This short, illustrative list of radio time signals contains information on time signals that are widely used and are controlled by observatories communicating their results to the Bureau International de l'Heure. Since transmission times and frequencies are liable to change current schedules should be consulted to obtain up-to-date information.

The International Astronomical Union (Dublin, 1955) has recommended the cessation of ONOGO and rhythmic type signals; details of such signals have therefore been excluded from this list.

Country	Authority	U.T.	Call Sign	Frequency kc/s	Notes
		h m			
Argentine	Naval Observatory, Buenos Aires	01 00 13 00 21 00	LOL	8 110 17 180	
	Military Geographical Institute, Buenos Aires	10 05 22 05	LQC	17 550	
Australia	Mount Stromlo Observatory, Canberra	08 00 14 00 20 00	VHP VIX	44 6 428·5 8 478 12 907·5	
Brazil	National Observatory, Rio de Janeiro	00 30 13 30 20 30	PPE	8 721	
		01 30 14 30 21 30	PPR	6 421 8 634 17 194	
Canada	Dominion Observatory, Ottawa		CHU	3 330 7 335 14 670	Continuous transmissions.
China	Zi-Ka-Wei Observatory, Shanghai	11 00 13 00 15 00	BPV	9 368	
France	Observatory of Paris	c8 00 09 00 09 30 13 00 20 00 21 00 22 30	FYP	91·15	
		09 00 21 00	FYA3	7 428	
		08 00 20 00	TQC9	10 775	
		09 30 13 00 22 30	TQG5	13 873	

continued on next page

* from the "Explanatory Supplement to the Ephemeris, 1961"

Table C-2 (continued)

Country	Authority	U.T. h m	Call Sign	Frequency kc/s	Notes
Germany (Federal German Republic)	German Hydrographic Institute, Hamburg	00 00	DAM	4 265 6 475·5 8 638·5	
		12 00	DAM	8 638·5 16 980	
		11 00	DMR20 DMR27	3 970 6 075	
		08 10 11 10	DCF77	77·5	
Germany (German Democratic Republic)	Geodetic Institute, Potsdam		DIZ	4 525	Continuous transmissions.
Japan	Astronomical Observatory, Tokyo	12 30	JAS22	16 170	
Switzerland	Cantonal Observatory, Neuchâtel	08 15	HBB	96·05	
Union of Soviet Socialist Republics	Central Scientific Investigation Institute, Moscow	00 00 04 00 08 00 12 00 16 00 20 00	ROR	25	
		08 00 to 22 00	RWM	5 000 10 000 15 000 20 000	Signals are transmitted on one or more of these frequencies at intervals of 2^h.
	Astronomical Observatory, Tashkent	18 00	RPT	5 890 11 580	
United Kingdom	Royal Greenwich Observatory, Herstmonceux	10 00 18 00	GBR GBZ	16·0 19·6	GBZ used as a reserve transmitter for GBR.
		10 00 18 00	GIC27 GIC29 GIC33 GIC37 GPB30 GKU5	7 397·5 9 350 13 555 17 685 10 332·5 12 790	Signals are transmitted on two of these frequencies at the times quoted.
United States of America	United States Naval Observatory, Washington	00 00 02 00 06 00 08 00 12 00 14 00 18 00 20 00	NSS	121·95 5 870 9 425 13 575 17 050 23 650	162 kc/s replaces 121·95 on transmissions at $18^h\ 00^m$ and $20^h\ 00^m$ on Tuesday, Wednesday, and Thursday. Transmissions are on all frequencies at the times quoted.
			NBA	18	Continuous transmissions except between $13^h\ 00^m$ and $21^h\ 00^m$ U.T. on Wednesday.

Table C-3 LIST OF COORDINATED TIME AND FREQUENCY TRANSMISSIONS *

This list contains information on the coordinated time and frequency transmissions of the United Kingdom and the United States of America. The co-operating authorities are: in the United Kingdom, the Royal Greenwich Observatory, the National Physical Laboratory, and the General Post Office; and in the United States, the U.S. Naval Observatory, the Naval Research Laboratory, and the National Bureau of Standards.

Country	Call Sign	Frequency kc/s	Transmission Times (U.T.)
United Kingdom	MSF	2 500	Continuous
		5 000	
		10 000	
		60	$14^h 30^m - 15^h 30^m$
	GBR	16	Continuous (traffic) except for daily maintenance between $13^h 00^m$ and $15^h 00^m$. Time signals at $10^h 00^m$ and $18^h 00^h$ only.
United States of America	WWV	2 500	Continuous
		5 000	
		10 000	
		15 000	
		20 000	
		25 000	
	WWVH	5 000	Continuous
		10 000	
		15 000	
	NBA	18	Continuous except between $13^h 00^m$ and $21^h 00^m$ on Wednesday.

* from the "Explanatory Supplement to the Ephemeris, 1961"

Table C-4 Constellation names and abbreviations *

The following list of constellation names and abbreviations is in accordance with the resolutions of the International Astronomical Union (*Trans. I.A.U.*, **1**, 158; **4**, 221; **9**, 66 and 77). The boundaries of the constellations are listed by E. Delporte, on behalf of the I.A.U., in *Délimitation scientifique des constellations (tables et cartes)*, Cambridge University Press, 1930; the areas of the constellations are given in *Handbook B.A.A.*, 1961.

Nominative		Genitive	Nominative		Genitive
Andromeda	And	Andromedae	Lacerta	Lac	Lacertae
Antlia	Ant	Antliae	Leo	Leo	Leonis
Apus	Aps	Apodis	Leo Minor	LMi	Leonis Minoris
Aquarius	Aqr	Aquarii	Lepus	Lep	Leporis
Aquila	Aql	Aquilae	Libra	Lib	Librae
Ara	Ara	Arae	Lupus	Lup	Lupi
*Argo	Arg	Argus	Lynx	Lyn	Lyncis
Aries	Ari	Arietis	Lyra	Lyr	Lyrae
Auriga	Aur	Aurigae	Mensa	Men	Mensae
Bootes	Boo	Bootis	Microscopium	Mic	Microscopii
Caelum	Cae	Caeli	Monoceros	Mon	Monocerotis
Camelopardalis	Cam	Camelopardalis	Musca	Mus	Muscae
Cancer	Cnc	Cancri	Norma	Nor	Normae
Canes Venatici	CVn	Canum Venaticorum	Octans	Oct	Octantis
Canis Major	CMa	Canis Majoris	Ophiuchus	Oph	Ophiuchi
Canis Minor	CMi	Canis Minoris	Orion	Ori	Orionis
Capricornus	Cap	Capricorni	Pavo	Pav	Pavonis
Carina	Car	Carinae	Pegasus	Peg	Pegasi
Cassiopeia	Cas	Cassiopeiae	Perseus	Per	Persei
Centaurus	Cen	Centauri	Phoenix	Phe	Phoenicis
Cepheus	Cep	Cephei	Pictor	Pic	Pictoris
Cetus	Cet	Ceti	Pisces	Psc	Piscium
Chamaeleon	Cha	Chamaeleontis	†Piscis Austrinus	PsA	Piscis Austrini
Circinus	Cir	Circini	Puppis	Pup	Puppis
Columba	Col	Columbae	Pyxis	Pyx	Pyxidis
Coma Berenices	Com	Comae Berenices	Reticulum	Ret	Reticuli
†Corona Austrina	CrA	Coronae Austrinae	Sagitta	Sge	Sagittae
Corona Borealis	CrB	Coronae Borealis	Sagittarius	Sgr	Sagittarii
Corvus	Crv	Corvi	Scorpius	Sco	Scorpii
Crater	Crt	Crateris	Sculptor	Scl	Sculptoris
Crux	Cru	Crucis	Scutum	Sct	Scuti
Cygnus	Cyg	Cygni	‡Serpens	Ser	Serpentis
Delphinus	Del	Delphini	Sextans	Sex	Sextantis
Dorado	Dor	Doradus	Taurus	Tau	Tauri
Draco	Dra	Draconis	Telescopium	Tel	Telescopii
Equuleus	Equ	Equulei	Triangulum	Tri	Trianguli
Eridanus	Eri	Eridani	Triangulum Australe	TrA	Trianguli Australis
Fornax	For	Fornacis	Tucana	Tuc	Tucanae
Gemini	Gem	Geminorum	Ursa Major	UMa	Ursae Majoris
Grus	Gru	Gruis	Ursa Minor	UMi	Ursae Minoris
Hercules	Her	Herculis	Vela	Vel	Velorum
Horologium	Hor	Horologii	Virgo	Vir	Virginis
Hydra	Hya	Hydrae	Volans	Vol	Volantis
Hydrus	Hyi	Hydri	Vulpecula	Vul	Vulpeculae
Indus	Ind	Indi			

* In modern usage Argo is divided into Carina, Puppis, and Vela.
† Australis is sometimes used, in both nominative and genitive.
‡ Serpens may be divided into Serpens Caput and Serpens Cauda.

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* from the "Explanatory Supplement to the Ephemeris, 1961"

Table C-5 Alphabetical star list

Popular Name	Astronomical Name	Approx. R.A., 1971
Aldebaran	α Tau	4^h 34^m
Alioth	ε UMa	12 52
Alkaid	η UMa	13 46
Al Nilam	ε Ori	5 34
Alphecca	α CrB	15 33
Alpheratz	α And	0 06
Altair	α Aql	19 49
Antares	α Sco	16 27
Arcturus	α Boo	14 14
Bellatrix	γ Ori	5 23
Betelgeuse	α Ori	5 53
Canopus	α Car	6 23
Capella	α Aur	5 14
Castor	α Gem	7 32
Deneb	α Cyg	20 40
Denebola	β Leo	11 47
Dubhe	α UMa	11 01
Elnath	β Tau	5 24
Eltanin (or Etamin)	γ Dra	17 55
Fomalhaut	α PsA	22 55
Hamal	α Ari	2 05
Kochab	β UMi	14 50
Markab	α Peg	23 03
Megrez	δ UMa	12 13
Merak	β UMa	10 59
Mizar	ζ UMa	13 22
Phecda	γ UMa	11 52
Polaris (the Pole Star)	α UMi	2 02
Pollux	β Gem	7 43
Procyon	α CMi	7 37
Regulus	α Leo	10 06
Rigel	β Ori	5 13
Saiph	κ Ori	5 46
Schedir	α Cas	0 38
Shaula	λ Sco	17 31
Sirius	α CMa	6 43
Spica	α Vir	13 23
Thuban	α Dra	14 03
Vega	α Lyr	18 35

REFERENCES

	Title	Author (or Editor)	Publisher
b	The Astronomical Universe	Krogdahl	MacMillan
b	Field Book of the Skies	Olcott & Mayall	Putnam
b	Practical Astronomy	Hosmer & Robbins	Wiley
b	Practical Astronomy	Nassau	McGraw-Hill
b	Sourcebook on the Space Sciences	Glasstone	Van Nostrand
b	Space Flight, vol. I; Environment & Celestial Mechanics	Ehricke	Van Nostrand
b	Spherical and Practical Astronomy as Applied to Geodesy	Mueller	Ungar
b	Spherical Astronomy	Woolard & Clemence	Academic Press
ba	The Astronomical Almanac	--	U. S. Government Printing Office
ba	The Nautical Almanac	--	"
b	Explanatory Supplement to the Ephemeris Green - out of print (almanac)	--	Her Majesty's Stationery Office, London
cp	Frequency and Time Standards (Application Note 52)	--	Hewlett-Packard Co. Palo Alto, Calif.
m	Sky & Telescope	--	Sky Publishing Co.

b = bound book
ba = bound book, annual volume
cp = company publication
m = magazine