Model of In-Plane Solar Irradiance for Front and Rear Side of PV Arrays

光伏方阵正反面辐照度的计算模型

Wang Sicheng  ERI, NDRC
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In-plane solar irradiance is the basic data to estimate PV power generation and useful in PV system designs:

**Battery Capacity Sizing**
蓄电池容量设计

**Balancing of Load Profile**
发电与负荷的平衡

**PV-Inverter Capacity Ratio**
光伏-逆变器容配比

In-plane solar irradiance can not be got from weather stations, they only have horizontal solar data. 气象局没有方阵面辐射量和辐照度的数据，只能提供水平面辐照度和辐射量的数据。
PV Arrays in Ground Horizontal Coordinates

The solar trackers are tracking the solar altitude and the solar azimuth by regulating the array tilted angle and array azimuth.
Ground Horizontal PV Arrays

- Fixed PV Array
- Solar Azimuth Trackers
- Manual Regulated Arrays
- Double Axis Trackers
PV Arrays in Equatorial Coordinates

The solar trackers are tracking the solar declination and the solar hour angle by regulating the array tilted angle and rotating angle of main axis.
Equatorial Tracking Systems

Horizontal E-W Tracking

Pole-Axis Tracking

Tilted E-W Tracking

Double Axis Tracking
3 Basic Rules and Concept


The triangle at celestial sphere

Arc: $a, b, c$
Angle: $A, B, C$

Cosine Rule:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
2. Cosine Rule of Direct Incidence (直接辐射的余弦定律)

\[ S_{T'} = S_D' \cos \theta \]
\[ S_H' = S_D' \cos \theta_Z = S_D' \sin \alpha \]
\[ S_D' = S_H' / \sin \alpha \]

So:
\[ S_{T'} = S_H' \cos \theta / \sin \alpha \]

\( S_{T'} \): solar irradiance on tilted array
\( S_D' \): direct solar irradiance
\( Z \): tilted angle of PV array
\( \theta \): incident angle of solar beam

\( S_{H'} \): solar irradiance on horizontal surface
\( \alpha \): solar altitude
\( \theta_Z \): zenith angle of solar beam
3. Formula of Solar Altitude $\alpha$

太阳高度角的公式能够在很多教科书上找到。

Can be found from any text book $^{[3,11]}$

$A$: Solar zenith angle $Za$, $Za = 90^\circ - \alpha$;
$B = 90^\circ - \varphi$;
$C = 90^\circ - \delta$;
$\omega$: hour angle. $a = \omega$

$$
\cos A = \cos B \cos C + \sin B \sin C \cos a \\
\cos A = \sin \alpha \\
\cos B = \sin \varphi \quad \sin B = \cos \varphi \\
\cos C = \sin \delta \quad \sin C = \cos \delta \\
\cos a = \cos \omega
$$

So: $\sin \alpha = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos \omega$
Solar Declination

太阳赤纬角

Cooper’s Formula:

\[ \delta = 23.5 \sin \left( \frac{360 \left( \frac{284 + N}{365} \right)}{360} \right) \]
How to get the Irradiance on PV front surface?

\[ Q'_T = S'_T + D'_T + R'_T \]

\[ S'_T = S'_H \times \frac{\cos \theta}{\sin \alpha} = S'_H \times R_b \]

\[ (R_b = \frac{\cos \theta}{\sin \alpha} \text{ is the ratio of tilted irradiance to the horizontal irradiance}) \]

\[ D'_T = D'_H \times \frac{1 + \cos Z'}{2} \]

\[ R'_T = \rho Q'_H \times \frac{1 - \cos Z'}{2} \]

5 variables are required for the calculation of in-plane total irradiance \( Q'_T \):

1) \( Q'_H \): Total irradiance on horizontal surface (kW/m2)
2) \( S'_H \): Direct irradiance on horizontal surface (kW/m2)
3) \( D'_H \): Diffuse irradiance on horizontal surface (kW/m2)
4) \( \cos \theta \): Cosine of solar incident angle
5) \( \cos Z' \): The instantaneous tilted angle of PV array
Diffuse Irradiance can be either Isotropic or Anisotropic

Isotropic model \(^{[1][2]}\) is for low irradiation and cloudy days:

\[
D'_T = D'_H (1 + \cos Z')/2 \quad \text{(by RetScreen)}
\]

For clear and sunny day, anisotropic model should be used: \(^{[6][8][9]}\)

\[
D'_T = D'_H \left( K(\cos \theta / \sin \alpha) + 1/2(1 + \cos Z')(1 - K) \right)
\]

anisotropic diffuse from circumsolar \hspace{1cm} isotropic sky diffuse

\[ K = S_H / Q_0 \] \hspace{1cm} K: the share of circumsolar diffuse which has the same characteristic of direct irradiance.
How to get Horizontal Irradiance Data?

\[ Q'_H = D'_H + S'_H \]

- \( Q'_H \): global irradiance at each hour (kW/m²)
- \( D'_H \): diffuse irradiance at each hour (kW/m²)
- \( S'_H \): direct irradiance at each hour (kW/m²)

1. **Multi-Year average real-tested hourly data**: can be found at Weather station or the database of NASA, PVsyst or Meteonorm database or NREL database. 多年平均实测数据（小时量）

2. **The horizontal hourly data can be got from Daily global and diffuse irradiation data** by the distribution models (Klein distribution [2,9,10] or Bouguer-Lambert distribution[4]). 从日总辐射量和日散射辐射量，通过日辐射分布模型得到。

3. **Daily Irradiation data can be got from Monthly Irradiation Data** by the way of interpolation [4]. 日辐射量可以通过插值从月辐射量得到。
\[
\cos \theta = \cos Z' \sin \alpha + \sin Z' \cos \alpha \cos (\beta - \gamma)
\]
cosθ for Equatorial H-E-W Tracking [Wang Sicheng]

Referenced by the formula of solar altitude α, we can get:

\[ \cos \theta = \sin \phi \sin \delta + \cos \phi \cos \delta \cos (\omega - \Omega) \]
General formula of $\cos \theta$
can be derived for Equatorial Coordinates

[Wang Sicheng]

Key Factor:

$b = 90^\circ - \varphi + Z + z$

\[
\cos a = \cos b \cos c + \sin b \sin c \cos A \\
\cos \theta = \cos(90^\circ - \varphi + Z + z)\cos \delta + \sin(90^\circ - \varphi + Z + z)\sin \delta \cos(\omega - \Omega)
\]
Instantaneous Tilted Angle of Array $Z'$ \[^{[3,11]}\]

Ground Horizontal Coordinates

(a) Fixed Array: $Z' = Z$; (it is the same with manual regulated arrays)

(b) Azimuth Tracking: $Z' = Z$

(c) Double Axis Tracking: $Z' = 90^\circ - \alpha$. 

Instantaneous Tilted Angle of Array $Z'$ for Equatorial Coordinates can be derived from the rule of spherical tringle. [4, Wang Sicheng]

For double-axis tracking in ground horizontal coordinates, we have:

$$Z' = 90^\circ - \alpha$$

If we set $z = \delta$ and $b = 90^\circ - z$

We will have:

$$\cos Z' = \sin \alpha$$

Then: $Z' = 90^\circ - \alpha$

(a) H-E-W Tracking: $Z = 0, z = 0, \cos Z' = \cos \Omega \quad Z' = \Omega$

(b) Tilted E-W Tracking: $Z = 0, z = z, \cos Z' = \cos z \cos \Omega$

(c) Pole-Axis Tracking: $Z = \varphi, z = 0, \cos Z' = \cos \varphi \cos \Omega$

(d) Double Axis Tracking: $Z = \varphi, z = -\delta, \Omega = \omega \quad \cos Z' = \sin(-\delta) \sin \varphi + \cos(-\delta) \cos \varphi \cos \Omega$
Now we have **all required formulas** for $\theta$ and $Z'$

For $\theta$:

Ground Horizontal Coordinates:

$$\cos \theta = \cos \theta \sin \alpha + \sin \theta \cos \alpha \cos (\beta - \gamma)$$

Equatorial Coordinates:

$$\cos \theta = \sin(90 - \phi + Z + z) \sin \delta + \cos (90 - \phi + Z + z) \cos \delta \cos (\omega - \Omega)$$

For $Z'$:

Ground Horizontal Coordinates: $Z'$ is always known.

Equatorial Coordinates:

$$\cos Z' = \sin z \sin Z + \cos z \cos Z \cos \Omega$$
The In-plane Solar Irradiance for Front-side

\[ Q'_T = S'_T + D'_T + R'_T \]

\[ S'_T = S'_D \cos \theta = S'_H R_b \quad (R_b = \cos \theta / \sin \alpha) \]

\[ D'_T = D'_H (1 + \cos Z')/2 \quad \text{(take diffuse irradiance as isotropic)} \]

\[ R'_T = \rho Q'_H (1 - \cos Z')/2 \]

The in-plane solar \textbf{daily irradiation} by integrating the irradiance from sunrise \((\omega_r)\) to sunset \((\omega_s)\):

\[ S_T = \int_{\omega_r}^{\omega_s} S'_T \, d\omega = \int_{\omega_r}^{\omega_s} S'_H \cos \theta / \sin \alpha \, d\omega \]

\[ D_T = \int_{\omega_r}^{\omega_s} D'_T \, d\omega = \int_{\omega_r}^{\omega_s} \frac{D'_H (1 + \cos Z')}{2} \, d\omega \]

\[ R_T = \int_{\omega_r}^{\omega_s} R'_T \, d\omega = \int_{\omega_r}^{\omega_s} \frac{\rho Q'_H (1 - \cos Z')}{2} \, d\omega \]

The monthly and yearly irradiation on PV array can be got simply sum-up the daily solar irradiations on PV array.
If we use anisotropic model for diffuse irradiance

\[ Q'_T = S'_T + D'_T + R'_T \]

\[ S'_T = S'_D \cos \theta = S'_H R_b \quad (R_b = \cos \theta / \sin \alpha) \]

\[ D'_T = D'_H \left[ \frac{S_H}{Q_0} R_b + \frac{1}{2} (1 - \frac{S_H}{Q_0})(1 + \cos Z') \right] \]

\[ R'_T = \rho Q'_H (1 - \cos Z') / 2 \]

And the **in-plane daily irradiation** by integrating the irradiance from sunrise \( (\omega_r) \) to sunset \( (\omega_s) \):

\[ S_T = \int_{\omega_r}^{\omega_s} S_T' \, d\omega = \int_{\omega_r}^{\omega_s} \frac{S_H' \cos \theta}{\sin \alpha} \, d\omega \]

\[ D_T = \int_{\omega_r}^{\omega_s} D_T' \, d\omega = \int_{\omega_r}^{\omega_s} D'_H \left[ \frac{S_H R_b}{Q_0} + \frac{1}{2} (1 - \frac{S_H}{Q_0})(1 + \cos Z') \right] \, d\omega \]

\[ R_T = \int_{\omega_r}^{\omega_s} R_T' \, d\omega = \int_{\omega_r}^{\omega_s} \rho Q'_H (1 - \cos Z') / 2 \, d\omega \]
How about Bifacial PV Modules?

How to calculate the irradiance on the rear side surface?
The Main Differences between Front side and Rear side

1. All irradiance received by front side and rear side: direct irradiance, diffuse irradiance (anisotropic circumsolar and isotropic sky diffuse), and reflected irradiance by the ground;
2. The models for direct and circumsolar diffuse on backside, and the sky diffuse irradiance on backside are the same as that for front side;
3. The main difference is the reflected irradiance.
Reference [1]

A Practical Irradiance Model for Bifacial PV Modules

Preprint

Bill Marion, Sara MacAlpine, and Chris Deline
National Renewable Energy Laboratory

Amir Asgharzadeh and Fatima Toor
University of Iowa

Daniel Riley, Joshua Stein, and Clifford Hansen
Sandia National Laboratories

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BSI = \( b \cdot F_b \cdot (DNI + I_{cir}) + \sum_{i=1}^{180^\circ} CF_i \cdot F_i \cdot I_i \) \hspace{1cm} (4)

where \( b \) = maximum (0, cosine of the AOI of the DNI); \( F_b \) is the AOI correction for the DNI using the air-glass model of Sjerps-Koomen et al. [10]; \( CF_i \) is the \( CF \) for the \( i \)th one-degree segment; \( F_i \) is the AOI correction for the \( i \)th one-degree segment; and \( I_i \) is the irradiance viewed by the \( i \)th one-degree segment (either \( I_{sky}, I_{hor}, \rho \cdot GRI_n, \) or \( I_{env} \)). The \( CF_i \) is represented by Eqn. 5:

\[
CF_i = \frac{1}{2} \cdot [\cos(i - 1) - \cos(i)]
\hspace{1cm} (5)
\]

where \( i \) is in degrees with a range from 1° to 180°. The field-of-view corresponding to a \( CF_i \) is shown in Fig. 2.

Fig. 2. Field-of-view of the ground for a one-degree segment depicted by the angles \( i \) and \( i-1 \).
German Paper: Model for Rear side Ground Reflection

4th International Conference on Silicon Photovoltaics, SiliconPV 2014

Simulation of energy production by bifacial modules with revision of ground reflection

Ufuk Alper Yusufoglu{$^a$}, Tae Hun Lee{$^a$}, Tobias Markus Pletzer{$^a$}, Andreas Halm{$^b$}, Lejo Joseph Koduvelikulathu{$^b$}, Corrado Comparotto{$^b$}, Radovan Kopecek{$^b$}, Heinrich Kurz{$^a$}

{$^a$}RWTH Aachen University, Institute of Semiconductor Electronics, Sommerfeldstraße 24, D-52074, Aachen, Germany
{$^b$}International Solar Energy Research Center Konstanz e.V., Rudolf-Diesel-Str. 15, D-78467, Konstanz, Germany

Key Points:
1. The beam and diffuse sky irradiance components received on the backside may be modeled with the same model used for the front side. This paper only study on reflected irradiance;
2. Assuming that the shadowing is caused only by the direct irradiance, and the reflected direct part of irradiance to the backside only from area outside of the shading.
3. The principle of View Factor (Fv) can be applied for the calculation of ground reflected irradiance at the module rear side.
4. The View Factor denotes the ratio of the irradiance reaching the back surface to the available irradiance on the ground.

Reference [2]
German Reference: the Backside Reflected Irradiance

The Reflected Irradiance to the backside:

\[ E_{POM, Albedo, rear} = \alpha DHI \frac{1 + \cos \beta}{2} + \alpha (GHI - DHI) \left( \frac{1 + \cos \beta}{2} - F_v \right) \]

View Factor (Fv)

Assuming the reflected direct part of irradiance to the rear side only from area outside of the shading.

Fig. 1. (a) Definition of view factor and; (b) its implementation for the ground reflected radiation.
Fraunhofer Developed a Ray Tracing Model

RAY TRACING

- 2nd approach: ray tracing
  - Tracing back the path of light: from the PV cell to the light source (= sun and diffuse) by taking into account its encounters with obstacles
  - Rays of light = straight lines
  - Diffuse and/or specular reflection
  - Example: one PV cell $\rightarrow$ 1 million of rays are sent by Monte Carlo, equiprobably distributed on the hemisphere $\rightarrow$ by successive reflections, they reach the light sources: sun and diffuse from the sky

Illustration of ray-tracing in 2D from one cell on the rear of a bifacial module
A large number of rays (only 80 are shown) are sent from the cell in every directions
Rays reaching the ground are randomly reflected

For a 3MWp plant, about 20 billions rays are sent, calculation time is about tens of minutes
The Main Factors to Affect the Irradiance on Rear Side

- Ground Albedo
- Module Number in Row (the row length)
- Ground Coverage Ratio (Distance between Arrays)
- Transparent Ratio of the modules
- Installation Height (Ground Clearance)
- Installation & Tilted Angle (Solar Trackers)
- Share of Daily Direct or Diffuse Irradiation
- Incidence Angle of Solar Beam
Assumptions for the model on rear side irradiance

1) the direct and isotropic sky diffuse irradiation on backside will follow the same models as that of front side;
2) the Direct and Circumsolar Diffuse part can only be reflected by the area without shading;
3) the installation height above the ground is high enough, so the non-uniformity on backside can be neglected, otherwise we need calculate the backside irradiance column by column (string by string) from bottom to top.
4) the installation height above the ground is high enough, so the transparent ratio does not further affect the backside irradiance. Or, we add a percentage of transparent ratio to the formulas;
5) The affect from the incident angle will be ignored, if calculating the power generation from backside, we can add a coefficient;
6) the reflected Direct part of irradiance will relay on the Shading Ratio or Shining Ratio, and not relevant to GCR.
We may define several View Factors

**F\(_{SR}\)**: denotes the shading ratio or shading factor to the total used land.

\[ F_{\text{SR}} = \text{shading area/total land usage of PV array (%)} \]

(How to get shading area and total land usage can be found in IEC/TR 63149-2018)

1-\(F_{\text{SR}}\): effective land ratio without shading (%)

\(\frac{1+\cos Z'}{2}\): sky view factor for front side \( F_{\text{SF}} \) (0 – 100%)

\(\frac{1-\cos Z'}{2}\): sky view factor for backside \( F_{\text{SB}} \) (0 – 100%)

\(\frac{1-\cos Z'}{2}\): ground view factor for front side \( F_{\text{GF}} \) (0 – 100%)

\(\frac{1+\cos Z'}{2}\): ground view factor for backside \( F_{\text{GB}} \) (0 – 100%)
The Key Issue is how to calculate the Shading Ratio? IEC/TR 63149

How about the front side? Never considered before?

- $F_{SR}$: Shading Ratio
- $K$: Array width
- $D_{shad-SN}$: shading distance, S-N direction
- $D_{shin-SN}$: shined distance under PV array, S-N direction
- $D_{rtr-SN}$: row to row distance, S-N direction
- $h_{A1}$: the array height
- $h_{A2}$: lowest point of array

\[ F_{SR} = \frac{K \times D_{shad-SN}}{K \times D_{rtr-SN}} = \frac{D_{shad-SN}}{D_{rtr-SN}} \]

\[ D_{shad-SN} = D1 + D2 \quad \text{(IEC/TR 63149)} \]

\[ D_{rtr-SN} \text{ is known (IEC/TR 63149)} \]

\[ D_{shin-SN} = \cos \beta \times \frac{h_{A2}}{\tan \alpha} \]
The pole is high and the shading is extended beyond $D_{\text{rtr-SN}}$.

Only the front row is different.
Formulas for rear side irradiance (1)

If we take **diffuse irradiance** as **isotropic**, we have:

\[ Q'_{TB} = S'_{TB} + D'_{TB} + R'_{TB} \]

\[ S'_{TB} = \cos \theta \quad S'_{DB} = S'_{HRb} \quad (R_b = \cos \theta / \sin \alpha) \]

(when \( \beta > 90^\circ \), \( S'_{TB} \) will shine on backside of fixed arrays)

\[ D'_{TB} = D'_H [1 + \cos(Z' + 180^\circ)] / 2 = D'_H (1 - \cos Z') / 2 \]

\[ R'_{TB} = \rho D'_H \left( \frac{1 + \cos Z'}{2} \right) + \rho (G'_H - D'_H) \left( \frac{1 + \cos Z'}{2} \right) (1 - F_v) \]

\[ = \rho D'_H \left( \frac{1 + \cos Z'}{2} \right) + \rho S'_H \left( \frac{1 + \cos Z'}{2} \right) (1 - F_v) \]

\[ G'_H = D'_H + S'_H \]
If we take diffuse irradiance as anisotropic, we have:

For the front side: \[ D’_T = D’_H \left[ \frac{S_H}{Q_0} R_b + \frac{1}{2} \left( 1 - \frac{S_H}{Q_0} \right) (1 + \cos Z') \right] \]

Circumsolar       Sky Isotropic Diffuse

For the backside: \[ D’_{TB} = D’_H \left[ \frac{S_H}{Q_0} R_b + \frac{1}{2} \left( 1 - \frac{S_H}{Q_0} \right) (1 - \cos Z') \right] \]

Circumsolar       Sky Isotropic Diffuse

\( S_H \): direct daily irradiation on horizontal surface
\( Q_0 \): is the extraterrestrial total daily solar irradiation on horizontal surface. [6]

So, we propose the rear side irradiance models as the followings:

\[ Q’_{TB} = S’_{TB} + D’_{TB} + R’_{TB} \]
\[ S’_{TB} + \text{Diffuse of Circumsolar} = S’_H R_b + D’_H \left( \frac{S_H}{Q_0} R_b \right) \]
\[ = [S’_H + D’_H \left( \frac{S_H}{Q_0} \right)] R_b \quad (R_b = \cos \theta / \sin \alpha) \quad (\text{when } \beta > 90^\circ) \]
\[ D’_{TB} = D’_H \left[ \frac{1}{2} \left( 1 - \frac{S_H}{Q_0} \right) (1 - \cos Z') \right] \]
\[ R’_{TB} = \rho \ D’_H \left[ \frac{1}{2} \left( 1 - \frac{S_H}{Q_0} \right) \left( \frac{1+\cos Z'}{2} \right) \right] + \rho \ (S’_H + D’_H \frac{S_H}{Q_0} \left( \frac{1+\cos Z'}{2} \right))(1 - Fv) \]
References for the Model of Front Side

References for the Model of Rear Side


[2] Ufuk Alper Yusufoglua, Tae Hun Leea (Germany), Simulation of energy production by bifacial modules with revision of ground reflection, 4th International Conference on Silicon Photovoltaics, SiliconPV 2014


[5] Paper by Solar World, Calculating the additional energy yield of bifacial solar modules,
